

# AIM Plate with Hole - Pre-Analysis

Author(s): Sebastian Vecchi, ANSYS Inc.

[Problem Specification](#)

[1. Pre-Analysis & Start-Up](#)

[2. Geometry](#)

[3. Mesh](#)

[4. Physics Setup](#)

[5. Results](#)

[6. Verification & Validation](#)

## Pre-Analysis & Start Up

### Analytical vs. Numerical Approaches

We can either assume the geometry as an infinite plate and solve the problem analytically, or approximate the geometry as a collection of "finite elements", and solve the problem numerically. The following flow chart compares the two approaches.

Let's first review the analytical results for the infinite plate. We'll then use these results to check the numerical solution from ANSYS.

### Analytical Results

#### Displacement

Let's estimate the expected displacement of the right edge relative to the center of the hole. We can get a reasonable estimate by neglecting the hole and approximating the entire plate as being in uniaxial tension. Dividing the applied tensile stress by the Young's modulus gives the uniform strain in the x direction.

Multiplying this by the half-width (5 in) gives the expected displacement of the right edge as ~ 0.1724 in. We'll check this against ANSYS AIM.

$r$

Let's consider the expected trends for  $r$ , the radial stress, in the vicinity of the hole and far from the hole. The analytical solution for  $r$  in an infinite plate is:

where  $a$  is the hole radius and  $\sigma$  is the applied uniform stress (denoted  $P$  in the problem specification). At the hole ( $r=a$ ), this reduces to

This result can be understood by looking at a vanishingly small element at the hole as shown schematically below.

We see that  $r$  at the hole is the normal stress at the hole. Since the hole is a free surface, this has to be zero.

For  $r \gg a$ ,

Far from the hole,  $r$  is a function of  $\theta$  only. At  $\theta = 0$ ,  $r \sim 0$ . This makes sense since  $r$  is aligned with  $x$  when  $\theta = 0$ . At  $\theta = 90^\circ$ ,  $r \sim 0$  which also makes sense since  $r$  is now aligned with  $y$ . We'll check these trends in the ANSYS AIM results.

Let's next consider the expected trends for  $\sigma_\theta$ , the circumferential stress, in the vicinity of the hole and far from the hole. The analytical solution for  $\sigma_\theta$  in an infinite plate is:

At  $r = a$ , this reduces to

At  $\theta = 90^\circ$ ,  $\sigma_\theta = 3\sigma_0$  for an infinite plate. This leads to a stress concentration factor of 3 for an infinite plate.

For  $r \gg a$ ,

At  $\theta = 0$  and  $\theta = 90^\circ$ , we get

Far from the hole,  $\sigma_x$  is a function of  $r$  only but its variation is the opposite of  $\sigma_y$  (which is not surprising since  $r$  and  $\theta$  are orthogonal coordinates; when  $r$  is aligned with  $x$ ,  $\theta$  is aligned with  $y$  and vice-versa). As one goes around the hole from  $\theta = 0$  to  $\theta = 90$  deg.,  $\sigma_x$  increases from 0 to  $\sigma_0$ . More trends to check in the ANSYS AIM results!

$\tau$

The analytical solution for the shear stress,  $\tau$ , in an infinite plate is:

At  $r = a$ ,

By looking at a vanishingly small element at the hole, we see that  $\tau$  is the shear stress on a stress surface, so it has to be zero.

For  $r \gg a$ ,

We can deduce that, far from the hole,  $\tau = 0$  both at  $\theta = 0$  and  $\theta = 90$  deg. Even more trends to check in ANSYS AIM!

$\sigma_x$

First, let's begin by finding the average stress, the nominal area stress, and the maximum stress with a concentration factor.

The concentration factor for an infinite plate with a hole is  $K = 3$ . The maximum stress for an infinite plate with a hole is:

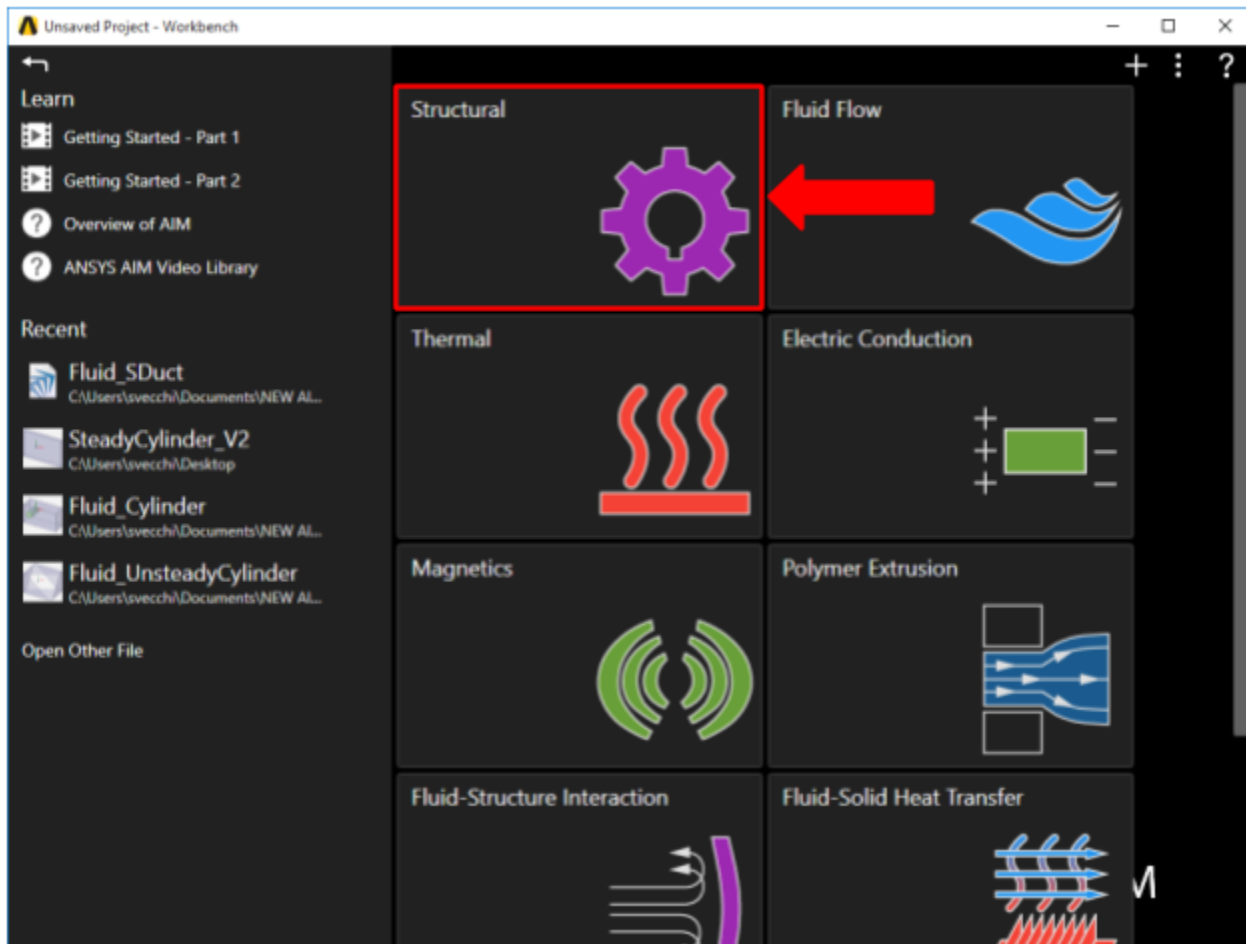
Although there is no analytical solution for a finite plate with a hole, there is empirical data available to find a concentration factor. Using a Concentration Factor Chart (Cornell 3250 Students: See Figure 4.22 on page 158 in Deformable Bodies and Their Material Behavior), we find that  $d/w = 1$  and thus  $K \sim 2.73$ . Now, we can find the maximum stress using the nominal stress and the concentration factor:

## Start-Up

A few words on the formatting on the following instructions:

1. Notes that require you to perform an action are colored in blue
2. General information is colored in black, but does not require any action
3. Words that are **bolded** are labels for items found in ANSYS AIM
4. Most important notes are colored in red

Now that we have the pre-calculations, we are ready begin simulating in ANSYS AIM. Open ANSYS AIM by going to **Start > All Apps > ANSYS 18.1 > ANSYS AIM 18.1**. Once you are at the start page of AIM, select the **Structural** template in the top left corner as shown below.



You will be prompted by the **Structural** Template to either **Define new geometry**, **Import geometry file**, or **Connect to active CAD session**. Select **Define new geometry** and press **Next**. For this problem we will be using static calculation type. Press **Finish** on the next panel. The Model Editor will launch automatically.

[Go to Step 2: Geometry](#)

[Go to all ANSYS AIM Learning Modules](#)