Particles in a Periodic Double Shear Flow - Pre-Analysis & Start-Up

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Pre-Analysis & Start-Up

In the *Pre-Analysis & Start-Up* step, we'll review the following:

- **Theory for Fluid Phase**
- **Theory for Particle Phase**
- **Choosing the Cases**

Pre-Analysis:

A particle laden flow is a multiphase flow where one phase is the fluid and the other is dispersed particles. Governing equations for both phases are implemented in Fluent. To run a meaningful simulation, a review of the theory is necessary.

Fluid Phase:

In the simulations considered for this tutorial, the fluid flow is a 2D perturbed periodic double shear layer as described in the first section. The geometry is Lx = 59.15m, Ly = 59.15m, and the mesh size is chosen as $\Delta x = L_x/n_x$ in order to resolve the smallest vorticies. As a rule of thumb. One typically needs about 20 grid points across the shear layers, where the vorticies are going to develop. The boundary conditions are periodic in the x and y directions. The fluid phase satisfies the Navier-Stokes Equations:
-Momentum Equations

$$\rho_f\left(\frac{d\mathbf{u}_f}{dt} + \mathbf{u}_f \cdot \nabla \mathbf{u}_f\right) = -\nabla p + \mu \nabla^2 \mathbf{u}_f + \mathbf{f}$$

-Continuity Equation

$$\frac{\partial \rho_f}{\partial t} + \nabla \cdot (\rho_f \mathbf{u}_f) = 0$$

where $\mathbf{u}$ is the fluid velocity, p the pressure, ρ_f the fluid density and $\mathbf{f}$ is a momentum exchange term due to the presence of particles. When the particle volume fraction α and the particle mass loading $M = \alpha\rho_p/\rho_f$ are very small, it is legitimate to neglect the effects of the particles on the fluid: $\mathbf{f}$ can be set to zero. This type of coupling is called one-way. In these simulations the fluid phase is air, while the dispersed phase is constituted of about 400 glass beads of diameter a few dozens of micron. This satisfies both conditions $\alpha \ll 1$ and $M \ll 1$

One way-coupling is legitimate here. See ANSYS documentation (16.2) for further details about the momentum exchange term.

Particle Phase:

The suspended particles are considered as rigid spheres of same diameter d, and density ρ_p. Newton's second law written for the particle i stipulates:

$$m_p \frac{d\mathbf{u}_p^i}{dt} = \mathbf{f}_{ext}^i$$

where $\mathbf{u}_p^i$ is the velocity of particle i, $\mathbf{f}_{ext}^i$ the forces exerted on it, and m_p its mass.
In order to know accurately the hydrodynamic forces exerted on a particle one needs to resolve the flow to a scale significantly smaller than the particle diameter. This is computationally prohibitive. Instead, the hydrodynamic forces can be approximated roughly to be proportional to the drift velocity ref3:

$$\frac{d\mathbf{u}_p^i}{dt} = \frac{\mathbf{u}_f - \mathbf{u}_p^i}{\tau_p}$$

where $\tau_p = \rho_p D^2/(18\mu)$ is known as the particle response time, ρ_p the particle density and D the particle diameter. This equation needs to be solved for all particles present in the domain. This is done in Fluent via the module: Discrete Phase Model(DPM).

Choosing the Cases:

The particle response time measures the speed at which the particle velocity adapts to the local flow speed. Non-inertial particles, or tracers, have a zero particle response time: they follow the fluid streamlines. Inertial particles with $\tau_p \neq 0$ might adapt quickly or slowly to the fluid speed variations depending on the relative variation of the flow and the particle response time.

This rate of adaptation is measured by a non-dimensional number called Stokes number representing the ratio of the particle response time to the flow characteristic time scale.

$$St = \frac{\tau_p}{\tau_f}$$

In these simulations, the characteristic flow time is the inverse of the growth rate of the vortices in the shear layers. This is also predicted by the Orr-Sommerfeld equation. For the particular geometry and configuration we used in this tutorial, the growth rate is $\sigma = 0.1751 s^{-1} = \frac{1}{\tau_f}$. When St = 0 the particles are tracers. They follow the streamlines and, in particular, they will not be able to leave a vortex once caught inside.

When $St \gg 1$, particles have a ballistic motion and are not affected by the local flow conditions. They are able to shoot through the vorticies without a strong trajectory deviation. Intermediate cases $St \approx 1$ have a maximum coupling between the two phases: particles are attracted to the vorticies, but once they reach the highly swirling vortex cores they are ejected due to their non zero inertia.

In this tutorial, we will consider a nearly tracer case St = 0.2, an intermediate case St = 1 and a nearly ballistic case St = 5.

Go to Step 2: Geometry

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