Hertz Contact Mechanics - Verification & Validation

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Verification and Validation

This section contains a few formulae, which made the listed assumptions, found in the *Pre-Analysis & Start-Up* page.

The analytical formula for computing the radius of contact zone (*a*) is given as follows:

$$a = \left(\frac{3F\left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right]}{4\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \right)^{1/3}$$

The following command for the computation of the contact area can be downloaded here.

- This command was generously provided by Mr. Sean Harvey. (Lead Technical Services Engineer at Ansys Inc.)

	Theoretical	Numerical	Relative Error (%)
Contact radius, a [mm]	1.00964	1.02517	1.538

Using this value of contact radius, we can also compute the normal pressured induced at the contact zone. Theoretically, the maximum pressure (p_{max}) is induced along the *y*-axis, as expected, and is given by the following formula:

$$p_{max} = \frac{3F}{2\pi a^2}$$

	Theoretical	Numerical	Relative Error (%)
Max. Pressure, p_{max} [MPa]	88.290	81.094	8.151

Furthermore, we can derive the following formula for the normal stresses $_z$ and $_r$ = along the *z*-axis.

$$\sigma_z = -p_{max}\left(\frac{z^2}{a^2}+1\right)^{-1}$$

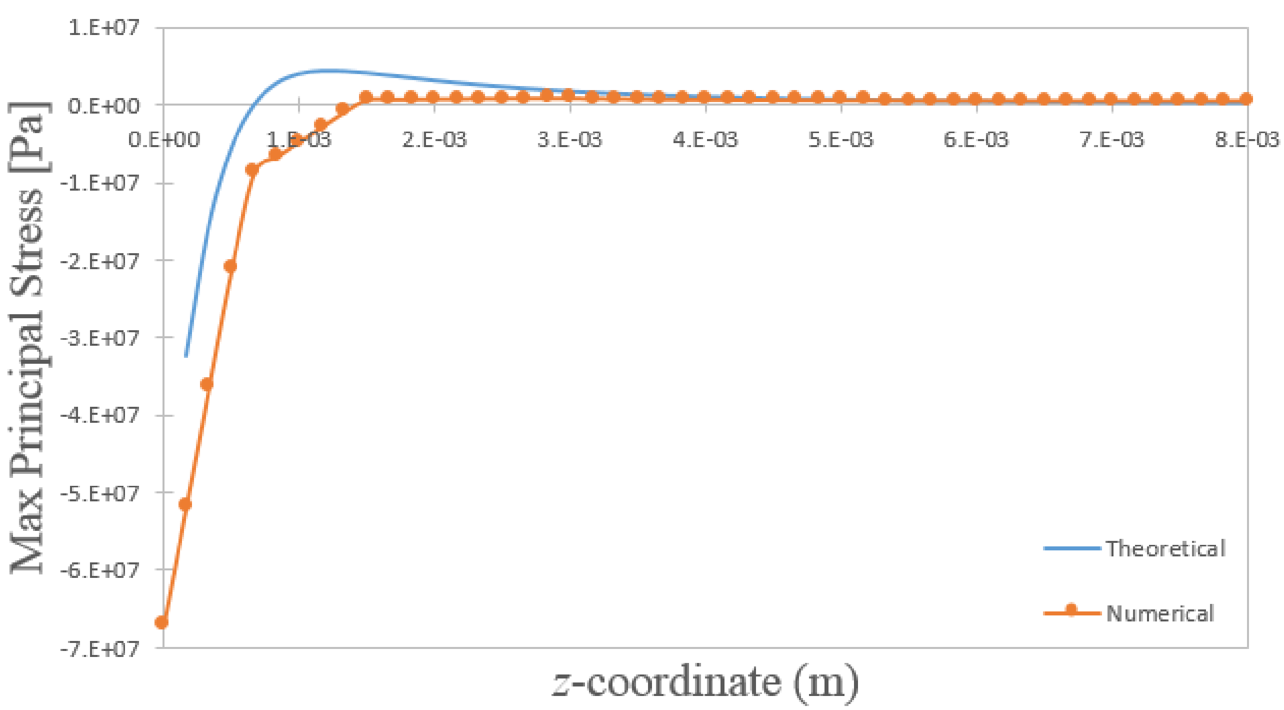
$$\sigma_r = \sigma_\theta = -p_{max}\left[(1+\nu_1)\left(1-\left|\frac{z}{a}\right|\tan^{-1}\left|\frac{a}{z}\right|\right)-\frac{1}{2\left(\frac{z^2}{a^2}+1\right)}\right]$$

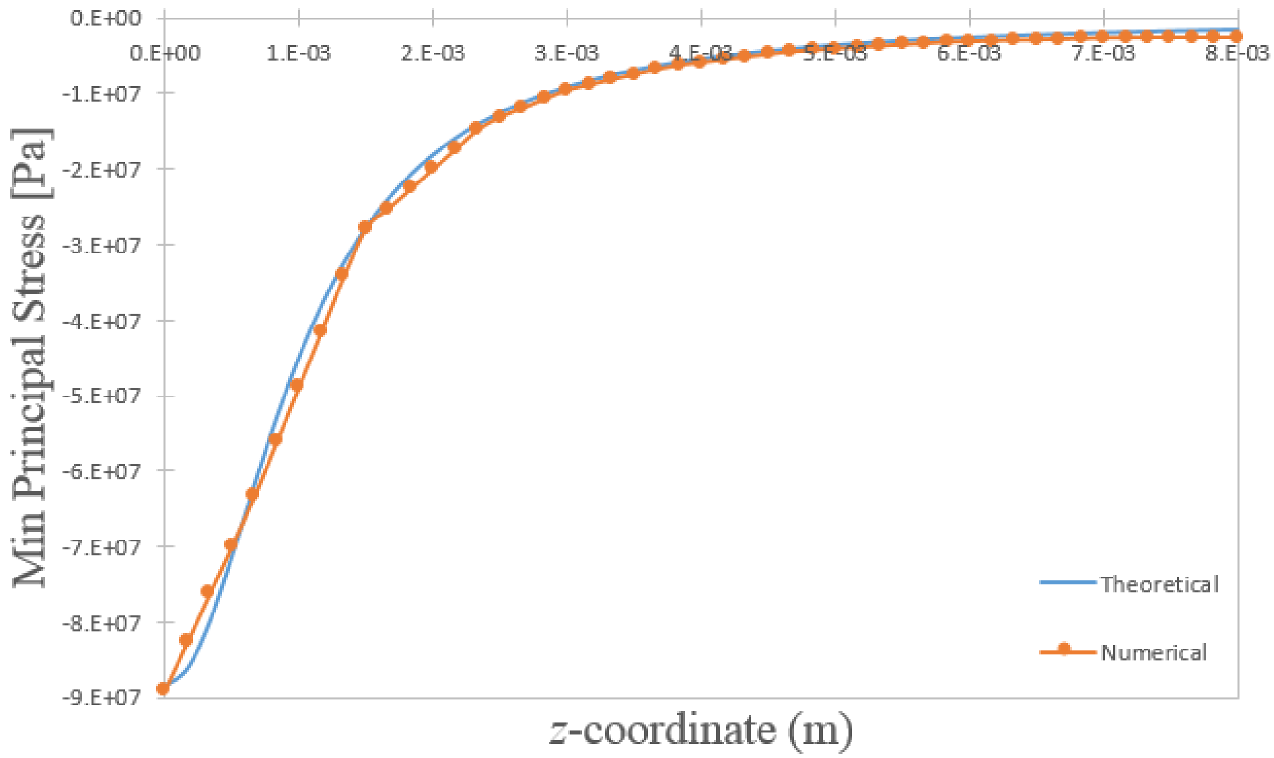
Here we note that the principal normal stresses $_1$ = $_2$ = $_r$ = since the *out-of-plane* shear stresses, $_{rz}$ = $_z$ = 0 and $_3$ = $_z$. And we can deduce that $_{max}$ = |$_1$|=|$_2$| =|($_1$-$_2$) / 2|. The effective stress (using the *Von-Mises criterion*) along the *y*-axis can be computed as the following:

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}$$

Lastly, we also confirm that the applied load at the top vertex of the sphere matches our numerical contact pressure, integrated along the interface.

Mesh size [m]	*2.00E-04*	*1.00E-04*	*9.00E-05*	*Theoretical*
Force Reaction (N)	187.95	188.32	188.52	188.50
Relative Error (%)	0.29	0.09	0.01	0.00





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