

# Hertz Contact Mechanics - Verification & Validation

Authors: Ju Hwan Shin and You Won Park

[Problem Specification](#)

[1. Pre-Analysis & Start-Up](#)

[2. Geometry](#)

[3. Mesh](#)

[4. Physics Setup](#)

[5. Numerical Solution](#)

[6. Numerical Results](#)

[7. Verification & Validation](#)

[Exercises](#)

[Comments](#)

## Verification and Validation

This section contains a few formulae, which made the listed assumptions, found in the *Pre-Analysis & Start-Up* page.

The analytical formula for computing the radius of contact zone ( $a$ ) is given as follows:

$$a = \left( \frac{3F \left[ \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right]}{4 \left( \frac{1}{R_1} + \frac{1}{R_2} \right)} \right)^{1/3}$$

The following command for the computation of the contact area can be downloaded [here](#).

- This command was generously provided by Mr. Sean Harvey. (Lead Technical Services Engineer at Ansys Inc.)

	Theoretical	Numerical	Relative Error (%)
Contact radius, $a$ [mm]	1.00964	1.02517	1.538

Using this value of contact radius, we can also compute the normal pressured induced at the contact zone. Theoretically, the maximum pressure ( $p_{\max}$ ) is induced along the  $y$ -axis, as expected, and is given by the following formula:

$$p_{\max} = \frac{3F}{2\pi a^2}$$

	Theoretical	Numerical	Relative Error (%)
Max. Pressure, $p_{\max}$ [MPa]	88.290	81.094	8.151

Furthermore, we can derive the following formula for the normal stresses  $\sigma_z$  and  $\tau_r =$  along the  $z$ -axis.

$$\sigma_z = -p_{max} \left( \frac{z^2}{a^2} + 1 \right)^{-1}$$

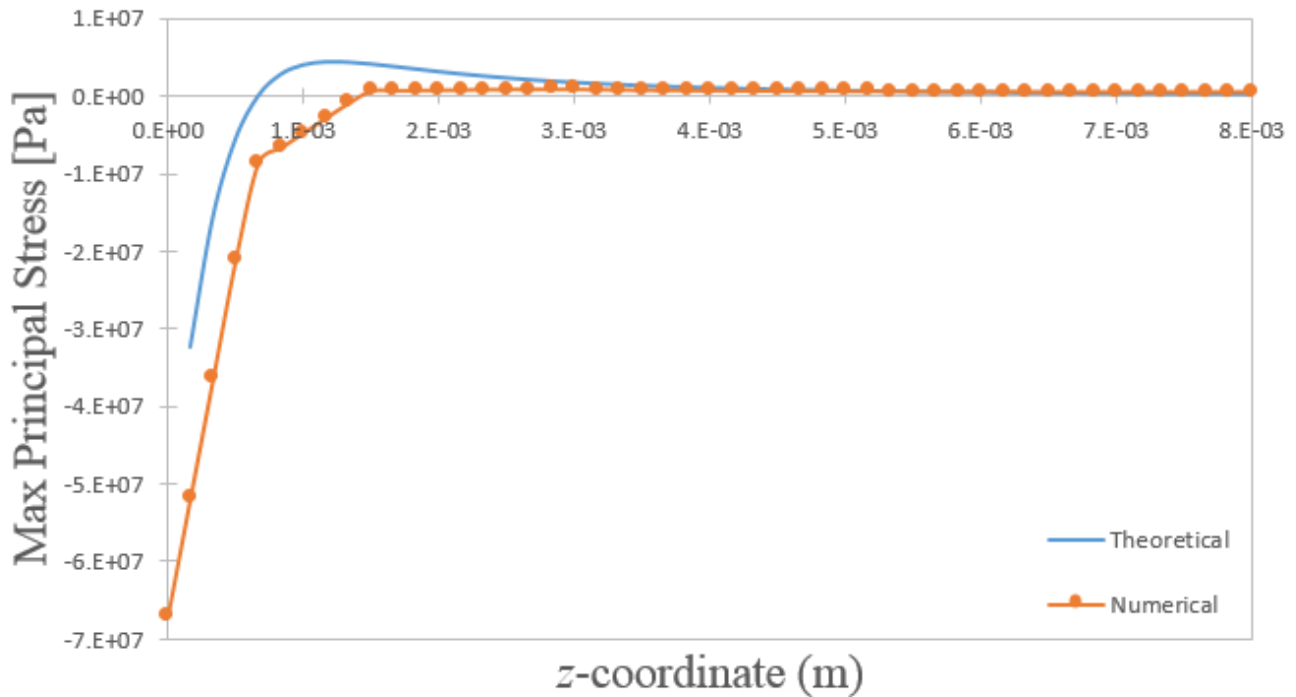
$$\sigma_r = \sigma_\theta = -p_{max} \left[ (1 + \nu_1) \left( 1 - \left| \frac{z}{a} \right| \tan^{-1} \left| \frac{a}{z} \right| \right) - \frac{1}{2 \left( \frac{z^2}{a^2} + 1 \right)} \right]$$

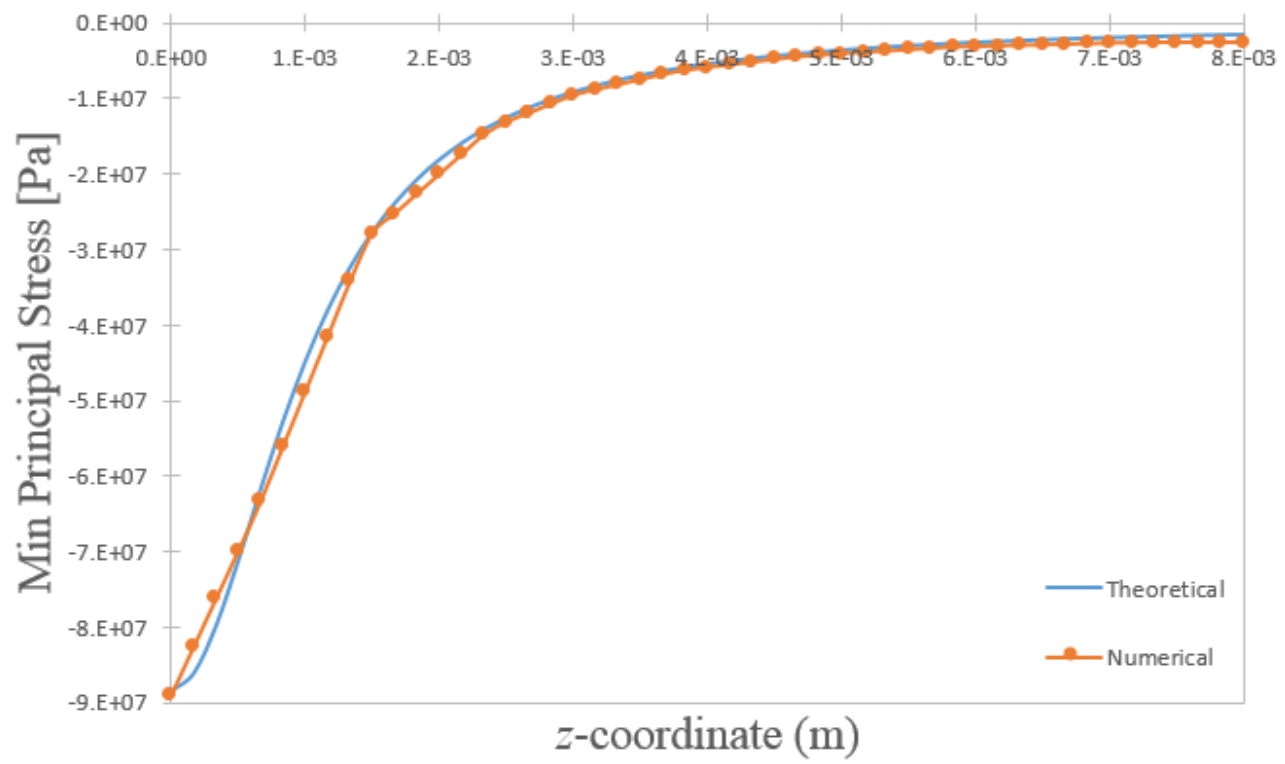
Here we note that the principal normal stresses  $\sigma_1 = \sigma_2 = \sigma_r$  since the *out-of-plane* shear stresses,  $\tau_{rz} = \tau_{zr} = 0$  and  $\sigma_3 = \sigma_z$ . And we can deduce that  $\sigma_{max} = |\sigma_1| = |\sigma_2| = |(\sigma_1 - \sigma_2) / 2|$ . The effective stress (using the *Von-Mises criterion*) along the *y*-axis can be computed as the following:

$$\bar{\sigma} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}$$

Lastly, we also confirm that the applied load at the top vertex of the sphere matches our numerical contact pressure, integrated along the interface.

<i>Mesh size [m]</i>	<i>2.00E-04</i>	<i>1.00E-04</i>	<i>9.00E-05</i>	<i>Theoretical</i>
<b>Force Reaction (N)</b>	187.95	188.32	188.52	188.50
<b>Relative Error (%)</b>	0.29	0.09	0.01	0.00





[Go to Exercises](#)

[Go to all ANSYS Learning Modules](#)