

Hertz Contact Mechanics - Verification & Validation

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Verification and Validation

This section contains a few formulae, which made the listed assumptions, found in the *Pre-Analysis & Start-Up* page.

The analytical formula for computing the radius of contact zone (a) is given as follows:

$$a = \left(\frac{3F \left[\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right]}{4 \left(\frac{1}{R_1} + \frac{1}{R_2} \right)} \right)^{1/3}$$

The following command for the computation of the contact area can be downloaded [here](#).

- This command was generously provided by Mr. Sean Harvey. (Lead Technical Services Engineer at Ansys Inc.)

	Theoretical	Numerical	Relative Error (%)
Contact radius, a [mm]	1.00964	1.02517	1.538

Using this value of contact radius, we can also compute the normal pressured induced at the contact zone. Theoretically, the maximum pressure ($p_{\max}$) is induced along the y -axis, as expected, and is given by the following formula:

$$p_{\max} = \frac{3F}{2\pi a^2}$$

	Theoretical	Numerical	Relative Error (%)
Max. Pressure, $p_{\max}$ [MPa]	88.290	81.094	8.151

Furthermore, we can derive the following formula for the normal stresses σ_z and $\tau_r =$ along the z -axis.

$$\sigma_z = -p_{max} \left(\frac{z^2}{a^2} + 1 \right)^{-1}$$

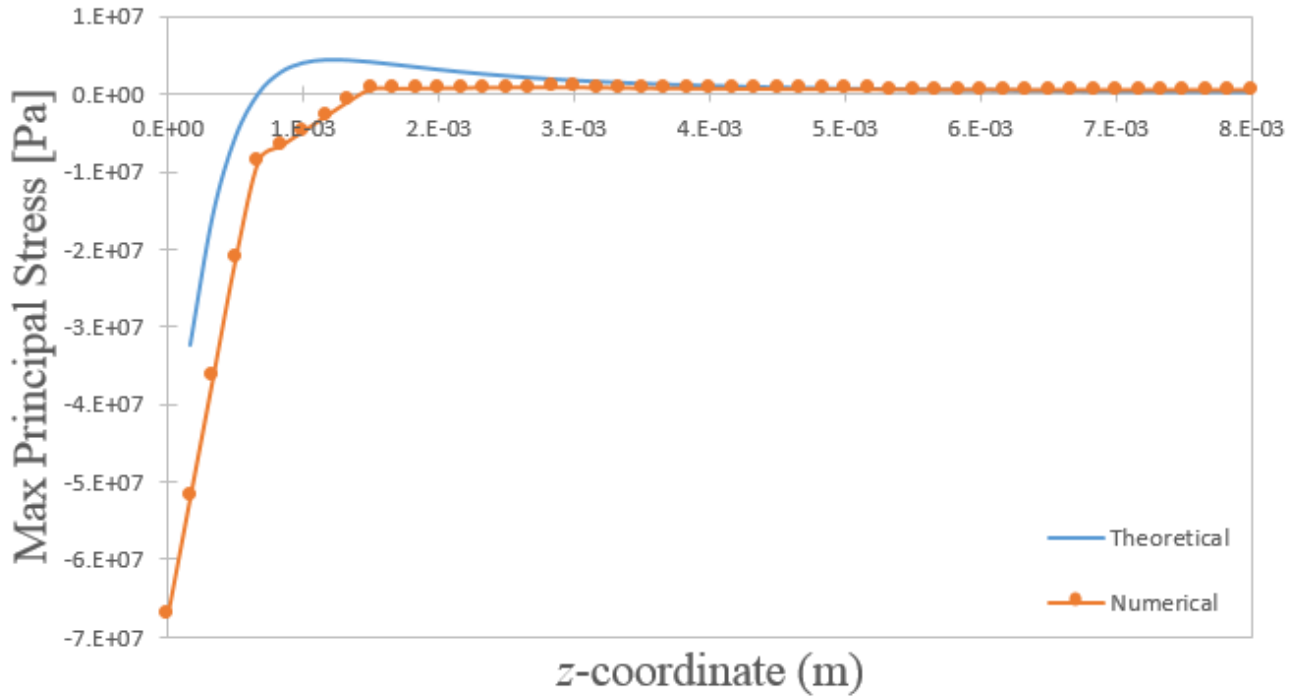
$$\sigma_r = \sigma_\theta = -p_{max} \left[(1 + \nu_1) \left(1 - \left| \frac{z}{a} \right| \tan^{-1} \left| \frac{a}{z} \right| \right) - \frac{1}{2 \left(\frac{z^2}{a^2} + 1 \right)} \right]$$

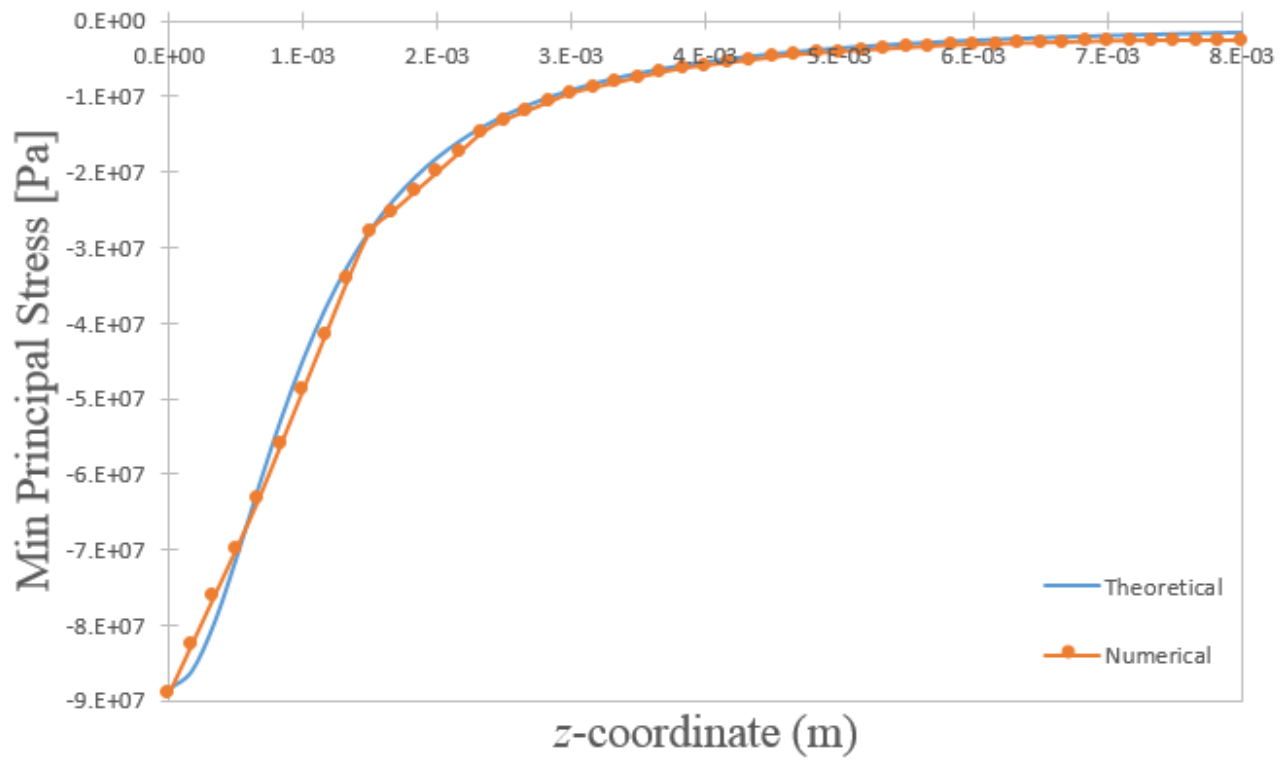
Here we note that the principal normal stresses $\sigma_1 = \sigma_2 = \sigma_r$ since the *out-of-plane* shear stresses, $\tau_{rz} = \tau_{zr} = 0$ and $\sigma_3 = \sigma_z$. And we can deduce that $\sigma_{max} = |\sigma_1| = |\sigma_2| = |(\sigma_1 - \sigma_2) / 2|$. The effective stress (using the *Von-Mises criterion*) along the y -axis can be computed as the following:

$$\bar{\sigma} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}$$

Lastly, we also confirm that the applied load at the top vertex of the sphere matches our numerical contact pressure, integrated along the interface.

<i>Mesh size [m]</i>	<i>2.00E-04</i>	<i>1.00E-04</i>	<i>9.00E-05</i>	<i>Theoretical</i>
Force Reaction (N)	187.95	188.32	188.52	188.50
Relative Error (%)	0.29	0.09	0.01	0.00





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