# Hertz Contact Mechanics - Verification & Validation

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## Verification and Validation

This section contains a few formulae, which made the listed assumptions, found in the *Pre-Analysis & Start-Up* page.

The analytical formula for computing the radius of contact zone (*a*) is given as follows:

$$a = \left( \frac{3F\left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right]}{4\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \right)^{1/3}$$

The following command for the computation of the contact area can be downloaded here.

- This command was generously provided by Mr. Sean Harvey. (Lead Technical Services Engineer at Ansys Inc.)

| | Theoretical | Numerical | Relative Error (%) |
|---|---|---|---|
| Contact radius, $a$ [mm] | 1.00964 | 1.02517 | 1.538 |

Using this value of contact radius, we can also compute the normal pressured induced at the contact zone. Theoretically, the maximum pressure ($p_{max}$) is induced along the *y*-axis, as expected, and is given by the following formula:

$$p_{max} = \frac{3F}{2\pi a^2}$$

| | Theoretical | Numerical | Relative Error (%) |
|---|---|---|---|
| Max. Pressure, $p_{max}$ [MPa] | 88.290 | 81.094 | 8.151 |

Furthermore, we can derive the following formula for the normal stresses $_z$ and $_r$ = along the *z*-axis.

$$\sigma_z = -p_{max}\left(\frac{z^2}{a^2}+1\right)^{-1}$$

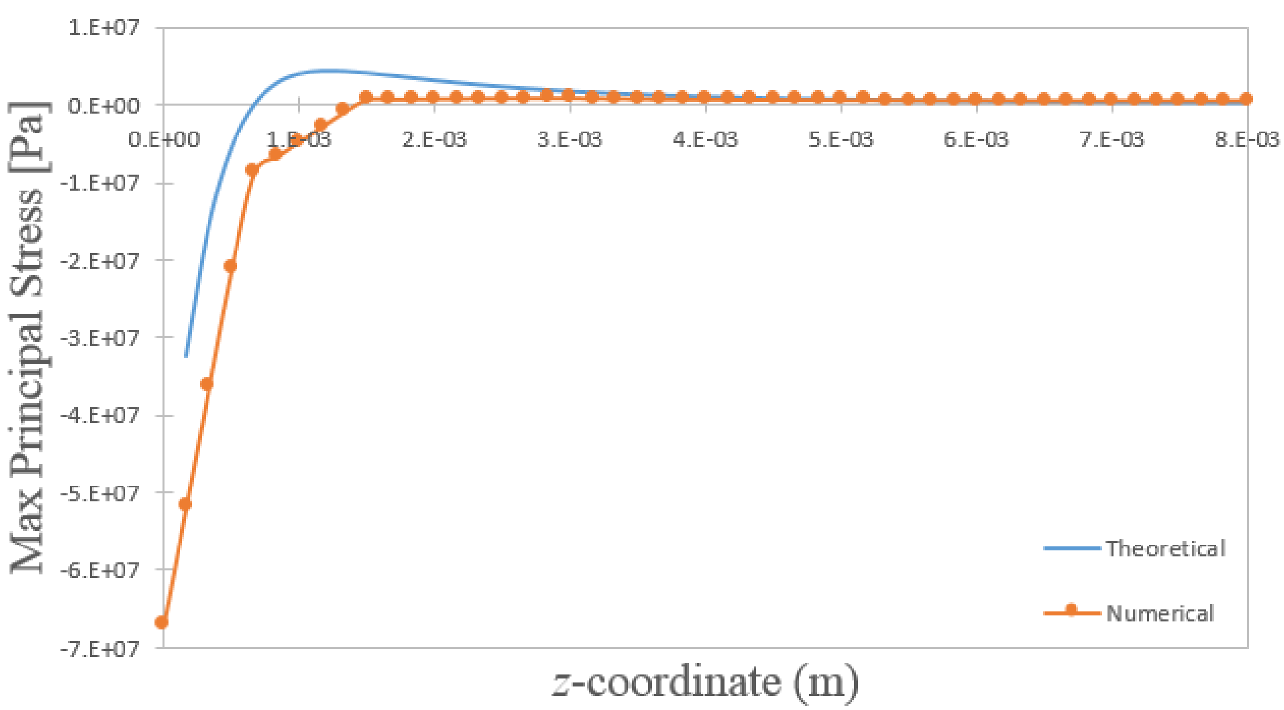
$$\sigma_r = \sigma_\theta = -p_{max}\left[(1+\nu_1)\left(1-\left|\frac{z}{a}\right|\tan^{-1}\left|\frac{a}{z}\right|\right)-\frac{1}{2\left(\frac{z^2}{a^2}+1\right)}\right]$$

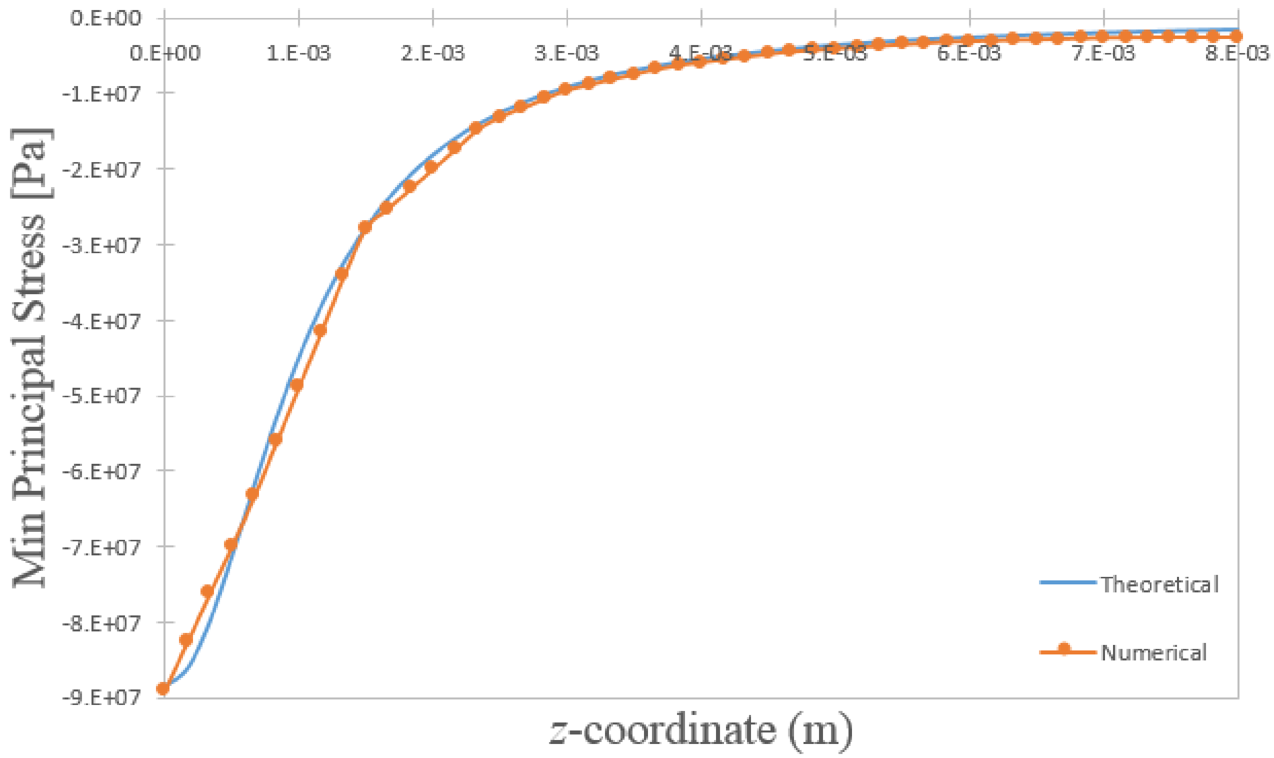
Here we note that the principal normal stresses $_1$ = $_2$ = $_r$ =  since the *out-of-plane* shear stresses, $_{rz}$ = $_z$ = 0 and $_3$ = $_z$. And we can deduce that $_{max}$ = |$_1$|=|$_2$| =|($_1$-$_2$) / 2|. The effective stress (using the *Von-Mises criterion*) along the *y*-axis can be computed as the following:

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}$$

Lastly, we also confirm that the applied load at the top vertex of the sphere matches our numerical contact pressure, integrated along the interface.

| ***Mesh size [m]*** | *2.00E-04* | *1.00E-04* | *9.00E-05* | *Theoretical* |
|---|---|---|---|---|
| **Force Reaction (N)** | 187.95 | 188.32 | 188.52 | 188.50 |
| **Relative Error (%)** | 0.29 | 0.09 | 0.01 | 0.00 |



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