Supersonic Flow Over a Wedge - Pre-Analysis & Start-Up

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Pre-Analysis & Start-Up

Pre-Analysis

In the hand calculations we will be applying the conservation of energy, mass and momentum equations for a 1D inviscid compressible flow. This differs from the way that FLUENT solves the problem as FLUENT instead uses the 2D inviscid compressible flow equations.

The equations can be written as:

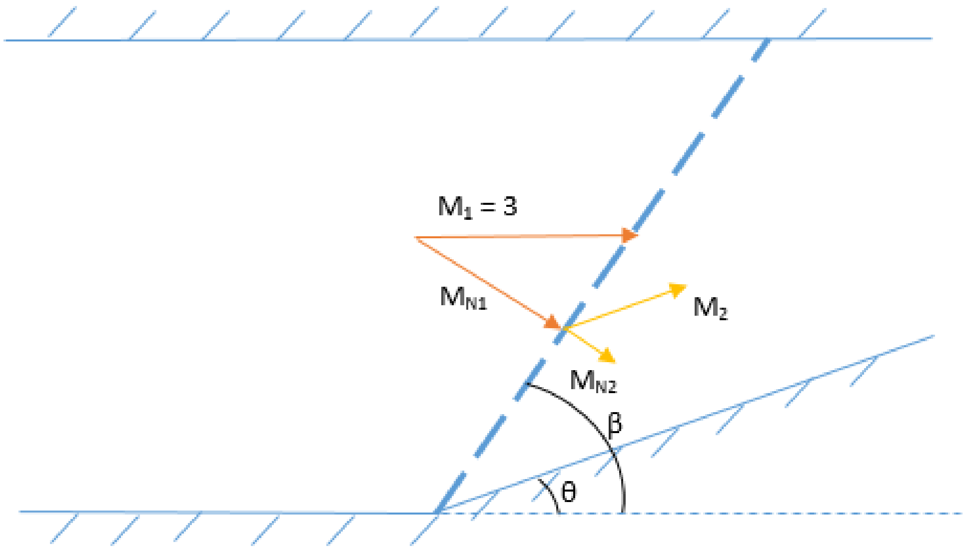
$$\frac{\partial e}{\partial t} + \mathbf{u} \cdot \nabla e + \frac{p}{\rho} \nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\nabla p}{\rho}$$

Hand Calculations

Flow with M = 3 comes straight on in the x-direction towards the wedge. We know the wedge angle theta from our geometry of the wedge to be 15 degrees. See the figure below:



Step 1: We then look at the Theta-Beta-M chart here we can find what the shock angle is corresponding to our conditions. The line M = 3 with wedge angle theta at 15 degrees corresponds to a shock angle beta of about 32 degrees.

Step 2: We calculate the value of the free stream Mach Number normal to the shock so we can use normal shock relations to relate quantities upstream and downstream of the shock.

$$M_{1N} = M_1 sin(\beta)$$

Step 3: Now we can relate the normal Mach numbers to each other through the normal shock relations

$$M_{2N}^2 = M_{1N}^2 \left(\frac{(\gamma - 1)M_{1N} + 2}{2\gamma M_{1N} - (\gamma - 1)} \right)$$

$$M_2 = \frac{M_{2N}}{sin(\beta - \theta)}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_{1N}^2 - (\gamma - 1)}{\gamma + 1}$$

$$\frac{T_2}{T_1} = \frac{(2\gamma M_{1N}^2 - (\gamma - 1))((\gamma - 1)M_{1N}^2 + 2)}{(\gamma + 1)^2 M_{1N}^2}$$

We expect that the flow downstream of the shock will still be supersonic as the flow experiences only a weak oblique shock, evident from looking at the theta-beta-M chart. This also becomes clear in the hand calculations.

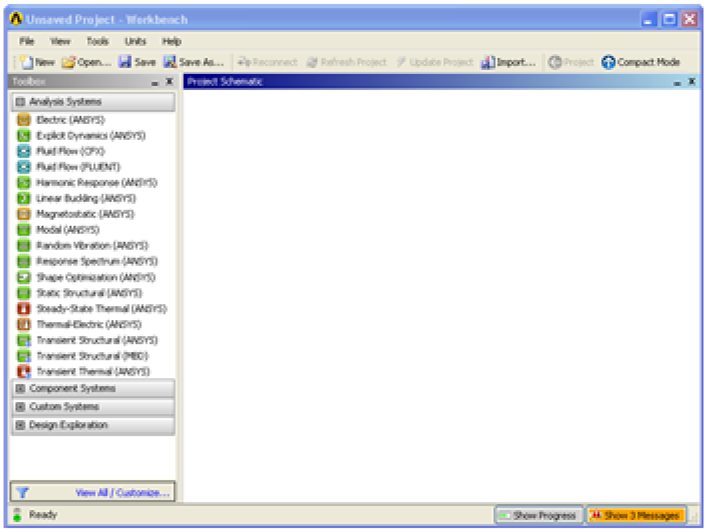
Alternate Procedure:

In order to calculate the expected results behind the shock, you can also use an oblique shock wave calculator (link grc.nasa.gov). At Mach 3 and an angle of 15 degrees, we find the following:

$$M_2 = 2.254$$
$$Shock\ Angle = 32.221^0$$
$$p_2 = 2.82 \text{ atmospheres}$$
$$T_2 = 416.4 \text{ k}$$

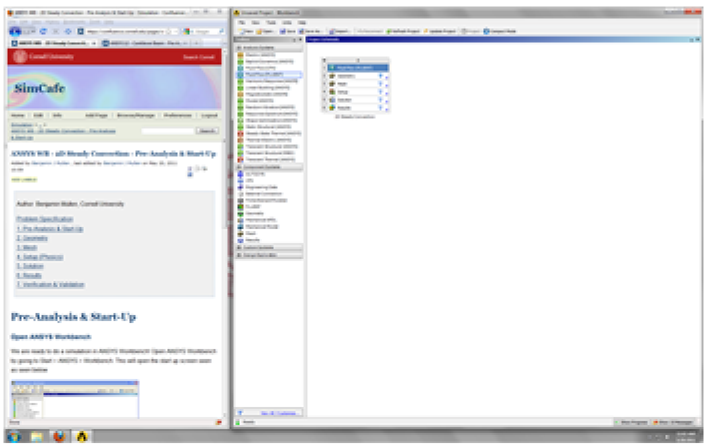
Open ANSYS Workbench

We are ready to do a simulation in ANSYS Workbench! Open ANSYS Workbench by going to Start > ANSYS > Workbench. This will open the start up screen seen as seen below:



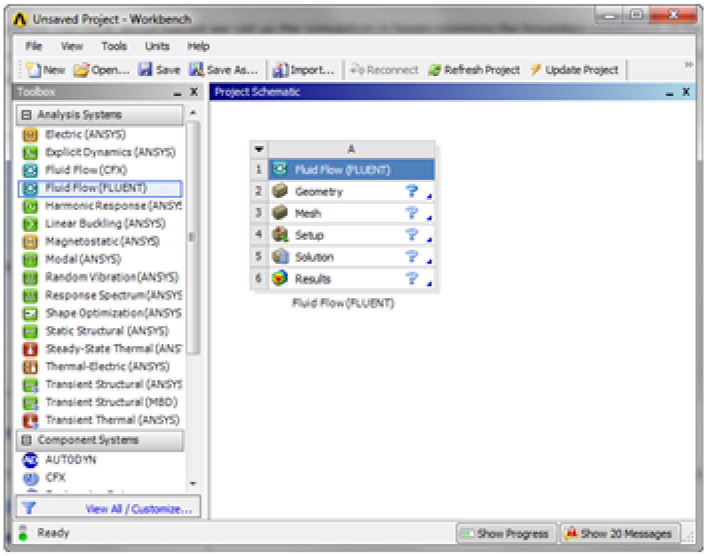
Screen Management

This tutorial is designed such that the user can have both ANSYS Workbench and the tutorial open. As shown below, this online tutorial should fill approximately 1/3 of the screen, while ANSYS Workbench fills the remaining 2/3 of the screen.



Setup Project

To begin, we need to tell ANSYS what kind of simulation we are doing. If you look to the left of the start up window, you will see the Toolbox Window. Take a look through the different selections. We will be using FLUENT to complete the simulation. Load the ***Fluid Flow (FLUENT)*** box by dragging and dropping it into the Project Schematic.



Right-click the top box of the project schematic 

and go to ***Rename***, and name the project `Supersonic Flow Over a Wedge`. You are ready to create the geometry for the simulation.

Go to Step 2: Geometry

Go to all FLUENT Learning Modules