Bending of a Curved Beam (Results-Interpretation) - Numerical Results

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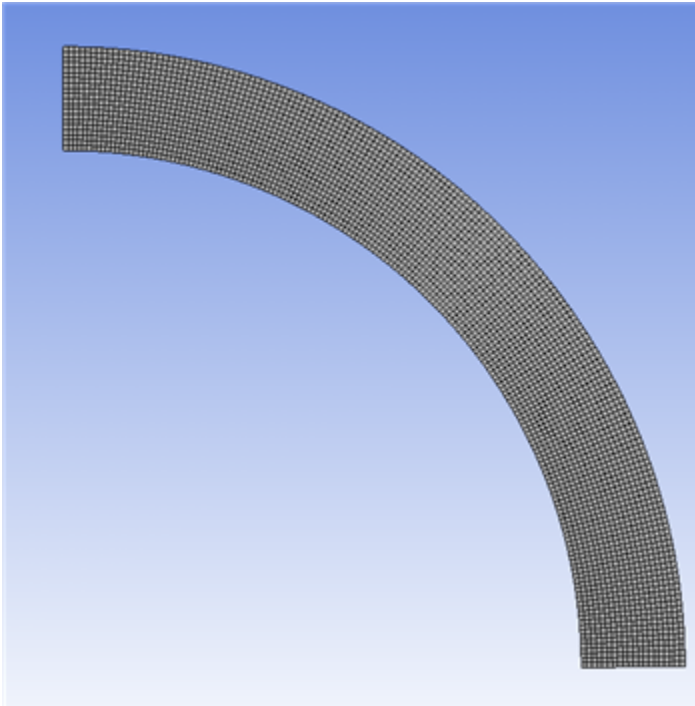## Numerical Results

Now we will examine the simulation results from ANSYS. *The ANSYS model uses the 2D plane stress approximation.*

Mesh

Before we dive in to the solution, let's take a look at the mesh used for the simulation. In the outline window, click **Mesh** to bring up the meshed geometry in the geometry window.

Only one-half of the geometry is modeled using symmetry constraints, which reduces the problem size. Look to the outline window under "Mesh". Notice that there are two types of meshing entities: a "mapped face meshing" and a "face sizing". The "mapped face meshing" is used to generate a regular mesh of quadrilaterals. The face sizing controls the size of the element edges in the 2D "face".

Displacement

Okay! Now we can check our solution. Let's start by examining how the beam deformed under the load. Before you start, make sure the software is working in the same units you are by looking to the menu bar and selecting **Units > US Customary (in, lbm, lbf, F, s, V, A)**. Now, look at the Outline window, and select **Solution > Total Deformation**.
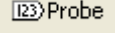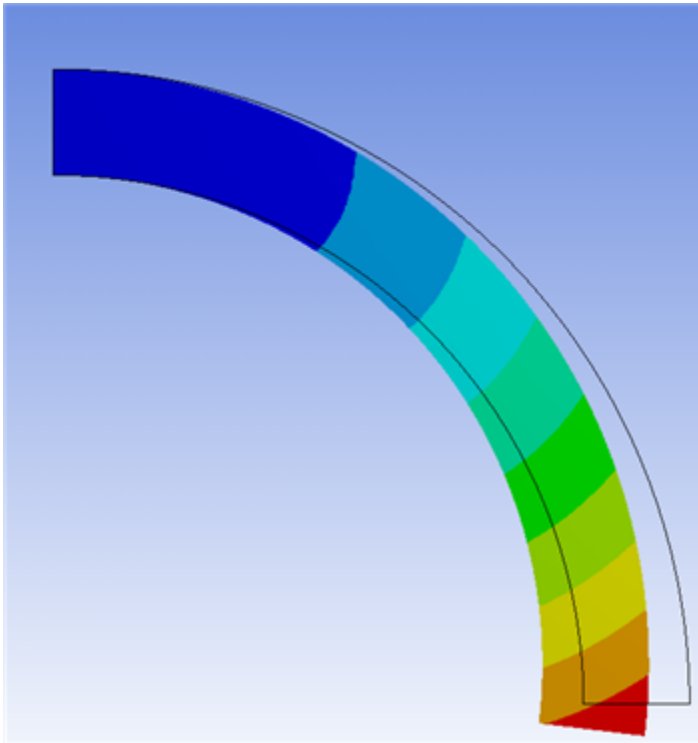
The colored section refers to the magnitude of the deformation (in inches) while the black outline is the undeformed geometry superimposed over the deformed model. The more red a section is, the more it has deformed while the more blue a section is, the less it has deformed. For this geometry, the bar is bending inward and the largest deformation occurs where the moment is applied , as one would intuitively expect.
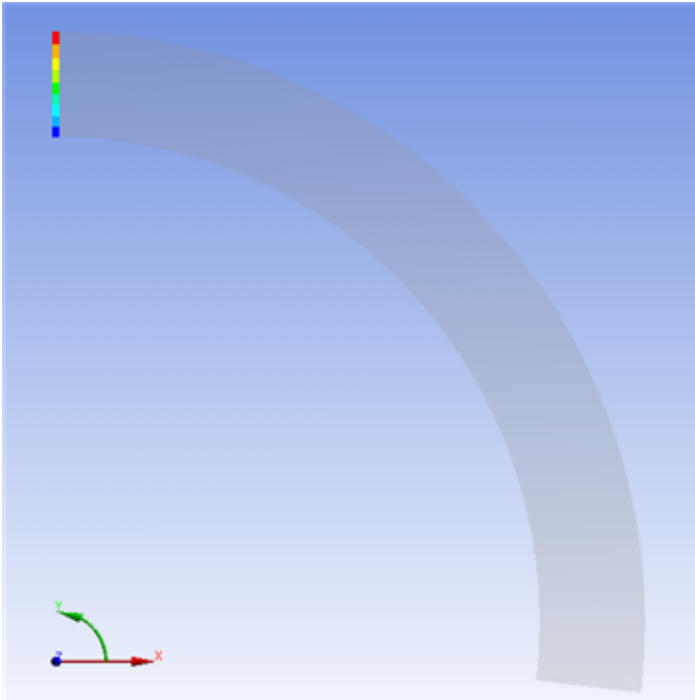
Sigma-theta

Click **Solution > Sigma-theta** in the outline window. This will bring up the distribution for the normal stress in the theta direction.
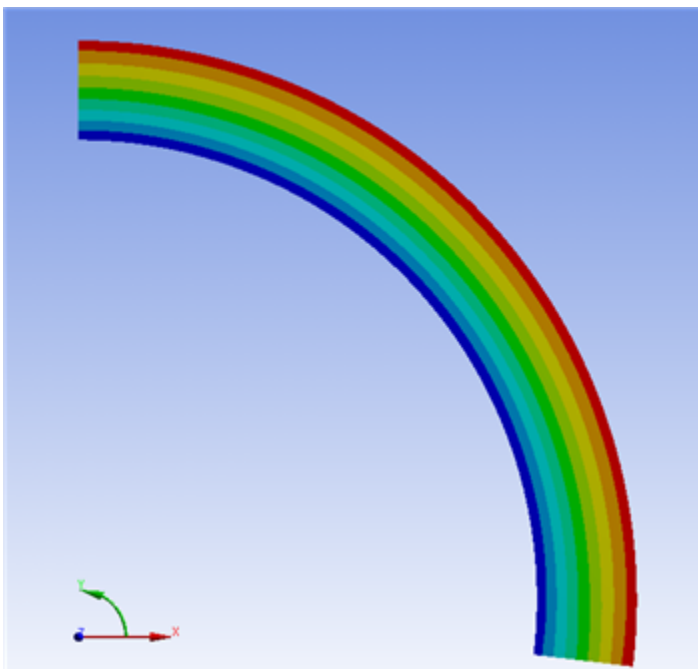
Sigma-theta, the bending stress, is a function of r only as expected from theory. It is tensile (positive) in the top part of the beam and compressive (negative) in the bottom part. There is a neutral axis that separates the tensile and compressive regions. The bending stress, Sigma-theta, is zero on the neutral surface. We will use the probe to locate the region where the bending stress changes from tensile to compressive. In order to find the neutral axis,

let's first enlarge the geometry. Do this by clicking the Box Zoom tool then click and drag a rectangle around the area you want to magnify. Now,

click the probe tool in the menu bar This will allow you to hover the cursor over the geometry to see the stress at that point. Hover the cursor

over the geometry until you have a good understanding of where the neutral axis on the beam is. To zoom out, click "Zoom to Fit"

We will now look at Sigma-theta along the symmetry line. Click **Solution > Sigma-theta along symmetry** in the outline window to bring up the stress distribution at the middle of the bar.


Look at the color bar to see the maximum and minimum stresses. The maximum theta-stress is 1697.63 psi and the minimum theta-stress is -1916.2 psi.

Sigma-r

In the outline window, click **Solution > Sigma-r**. This will bring up the distribution for the normal stress in the r-direction.
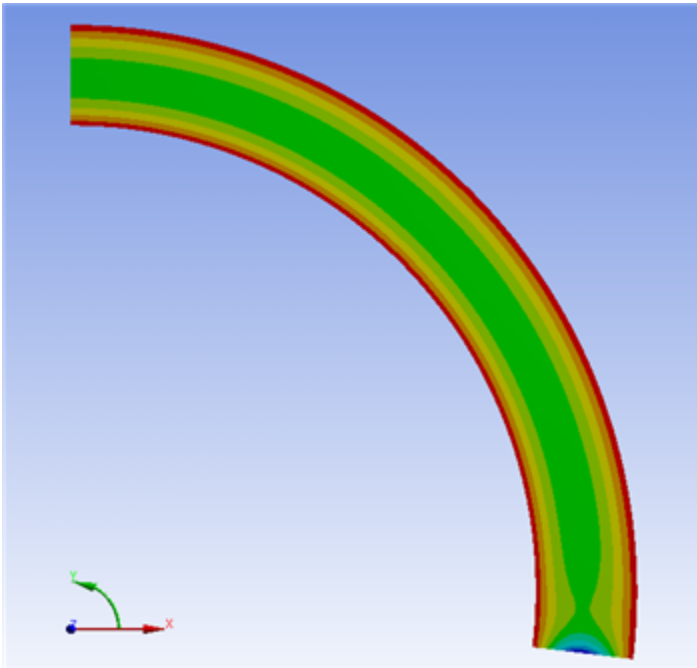
Looking at the distribution, we can see that the stress varies only as a function of r as expected. The magnitude of Sigma-r is much lower than Sigma-theta (this is why Winkler-Bach theory assumes Sigma-r =0). Also, we can see that there is a stress concentration in the area where the moment is applied. In the theory, this effect is ignored. In order to further examine the Sigma-r, let's look at the variation along the symmetry line. Click on *Solution > Sigma-r along symmetry*. This solution is the normal stress in the r-direction at the midsection of the beam.
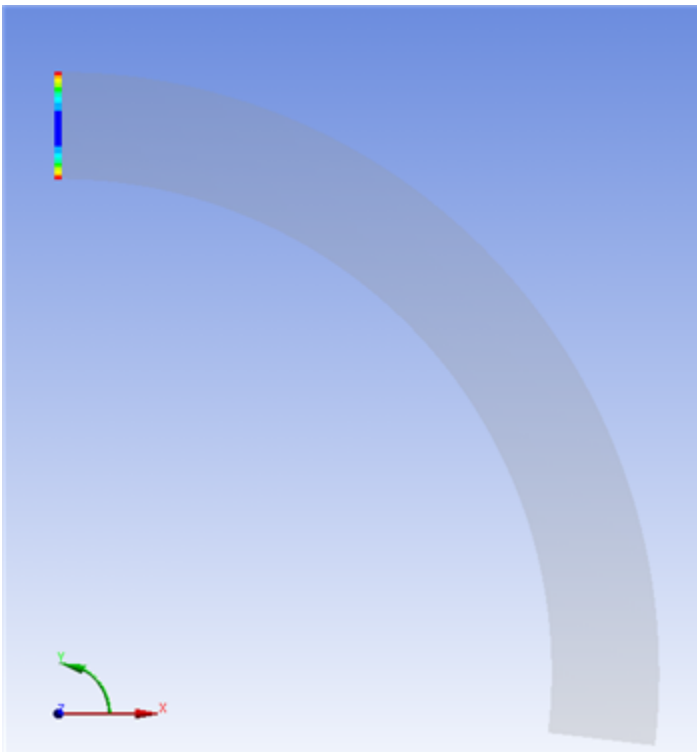
Looking at the color bar again, we can see that the maximum r-stress is -.110 psi, and the minimum r-stress is -82.302 psi. At r=a and r=b, Sigma-r ~ 0 as one would expect for a free surface.

Tau-r-theta

In the details window, click *Solution > Tau-r-theta* to bring up the stress distribution for shear stress.
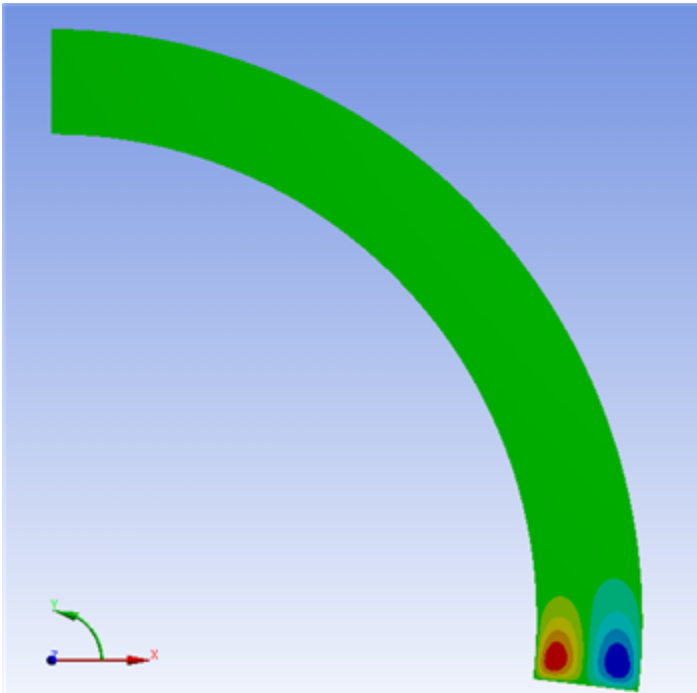
Hover the probe tool over points on the geometry far from the moment. You will notice that the stress is on the order of 10e-7. For a beam in pure bending, we assume that the shear stress is zero. However, ANSYS does not make this assumption: it calculates a value for shear stress at every point on the beam. Therefore, it is reassuring that the shear stress is almost negligible, which reinforces our assumption that it is zero.

Solution at r = 11.5 Inches

Now that we have a good idea about the stress distribution, we will look specifically at solving the problem in the problem specification. First, we will look at the stress in the r-direction at r = 11.5 inches. In the outline window, click **Solution > Sigma-r at r =11.5**. This will bring up the stress in the r-direction along the path at r = 11.5 inches (from the center of curvature of the bar).
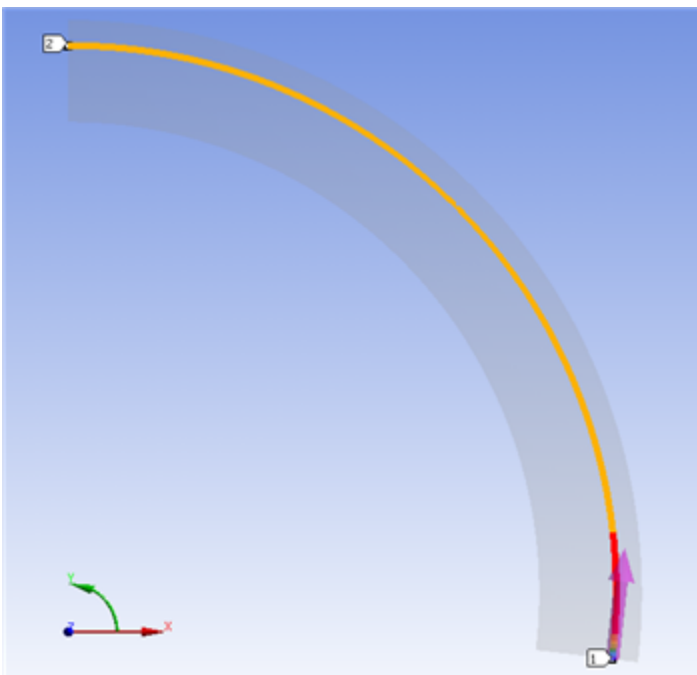

In the window below, there is a table of the stress values along the path. To find the value of sigma-r at r = 11.5 in, we want to look far away from the stress concentration region due to the moment. The path is defined in a counter-clockwise direction, so looking at the last value of the table should tell us the stress at r = 11.5 inches at the midsection of the bar. This value of sigma-r is -57.042 psi.

Now, we will do the same for the stress in theta direction to determine sigma-theta at r = 11.5 inches. In the outline window, click **Solution > Sigma-theta at r =11.5**. This will bring up the stress in the theta-direction along the path at r 11.5 inches.
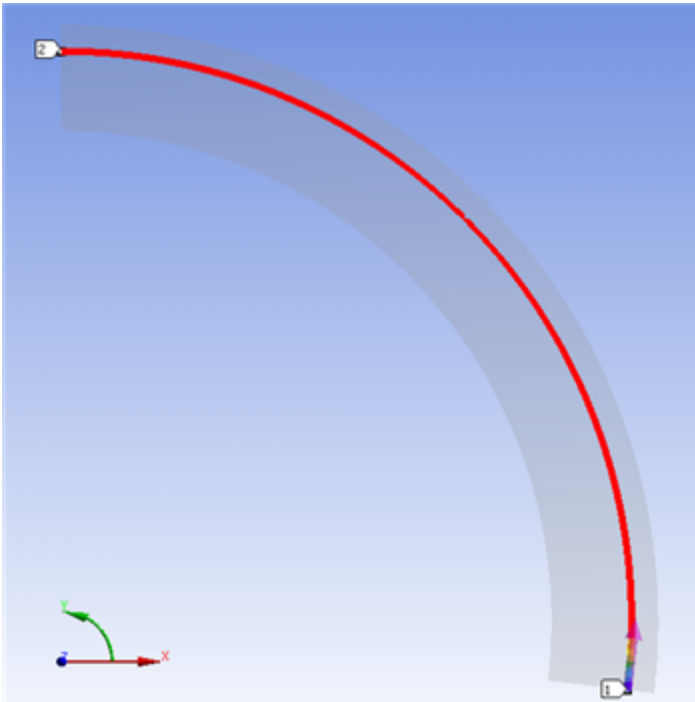
Look again at the table containing the stresses along the path. Look to the bottom of the table to find the stress in the theta-direction at the midpoint of the bar. We find that sigma-theta at this point is 910.950 psi. Compare this to what you would expect from curved beam theory.

Finally, we will examine the shear stress at r = 11.5 in. In the outline window, click **Solution > Tau-r-theta at r =11.5**.
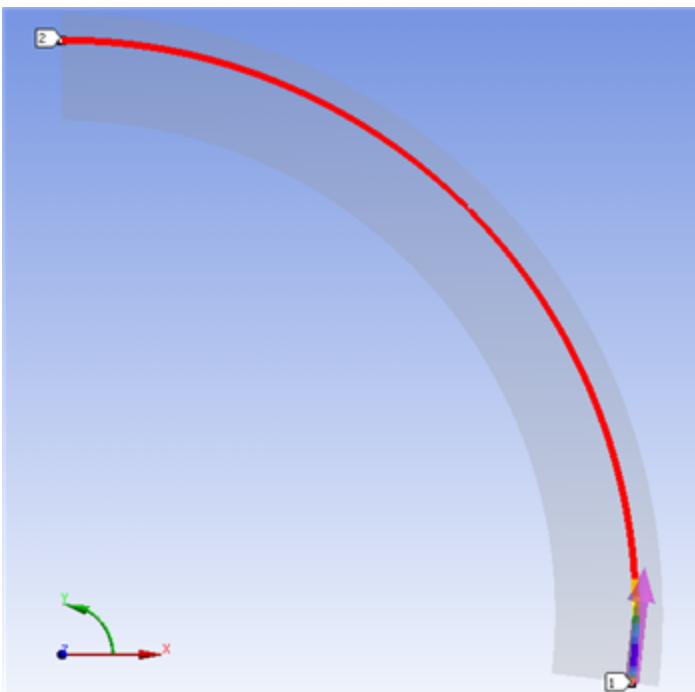
Again, look at the bottom of the table. You will find that the shear stress is very small at this point as we mentioned above.

Comparison.

Now that we have our results from the ANSYS simulation, let's compare them to the theory calculations. Below is chart comparing the values found in ANSYS, and through calculations using the Elasticity Theory, Winkler-Bach Theory, and Straight Beam Theory (Note: all stress values are in psi)

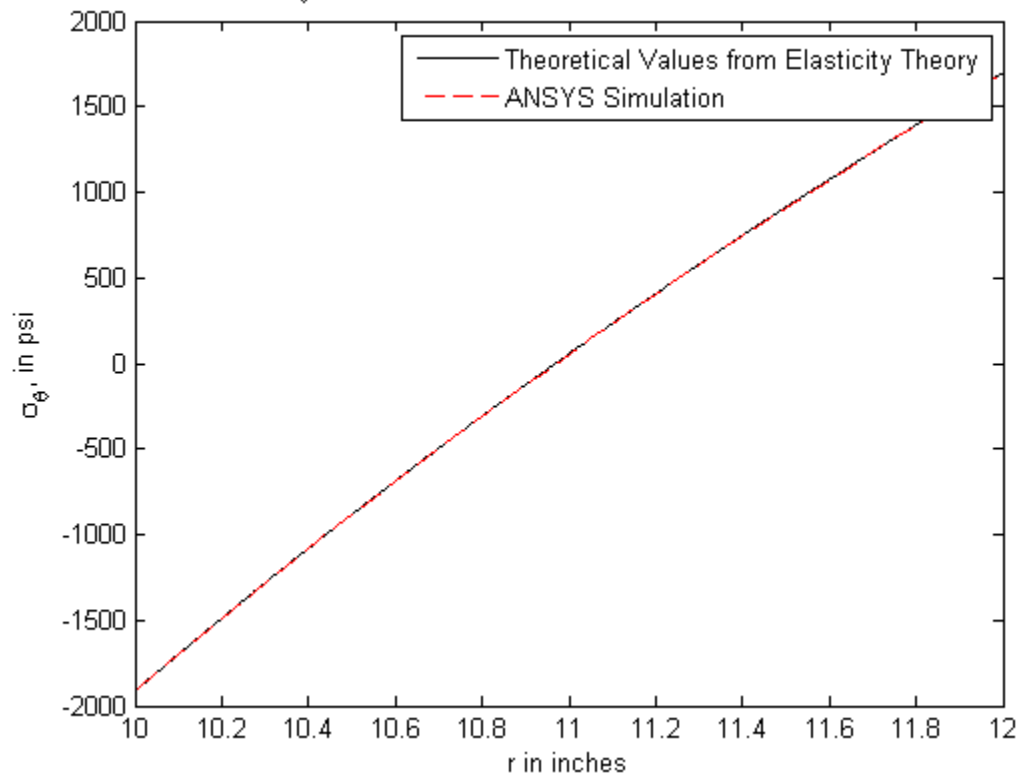|	Elastic Theory	Winkler Bach Theory	Straight Beam Theory	ANSYS	Percent Difference (Elastic Theory)	Percent Difference (Winkler Bach Theory)	Percent Difference Beam Theory
Maximum Sigma_r	0.00	N/A	0	-0.11	N/A	N/A	N/A
Minimum Sigma_r	-82.21	N/A	0	-82.30	0.11%	N/A	N/A
Maximum Sigma_theta	1697.00	1696.40	1800	1697.63	0.04%	0.07%	5.85%
Minimum Sigma_theta	-1917.00	-1915.60	-1800	-1916.20	0.04%	0.03%	6.25%
Sigma_r at r = 11.5	-56.93	N/A	0	-57.04	0.20%	N/A	N/A
Sigma_theta at r = 11.5	910.70	911.14	900	910.95	0.03%	0.02%	1.21%

Now, let's see how the stress distributions vary along the beam for each theory.

First, let's see how the Elastic Theory compared to the ANSYS solution (you can right-click on the tabular data and export it in Excel or Text format to make the plots below):

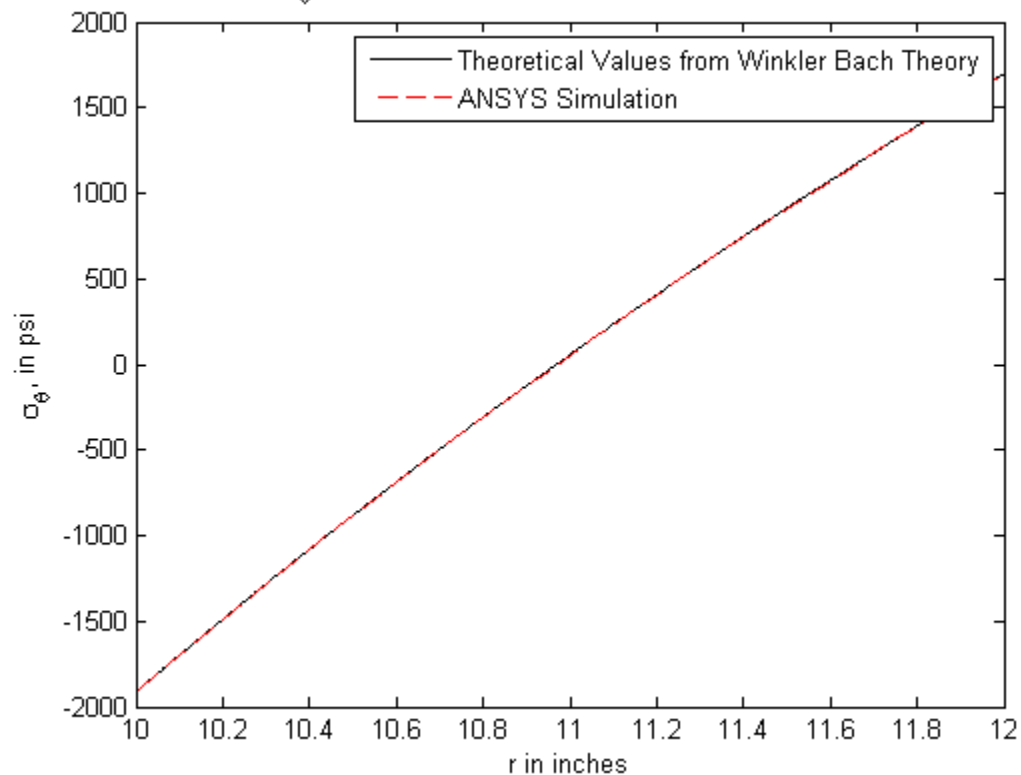## Stress Distribution for σ_r Along a Curved Beam a = 10 in, b = 12 in, t = .25 in, M = 300 lbf/in²
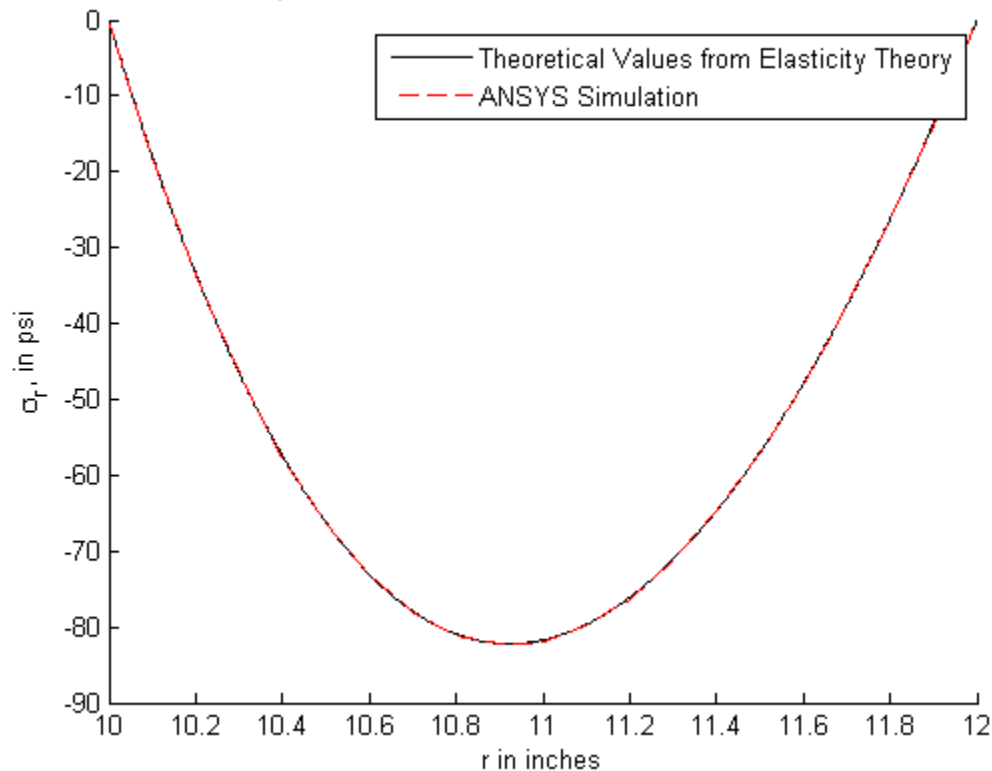
Stress Distribution for σ_θ Along a Curved Beam a = 10 in, b = 12 in, t = .25 in, M = 300 lbf/in²


From what we can see from the able graphs, the Elastic Theory matched the ANSYS solution very well. The same can be said for the Winkler-Bach theory:

Stress Distribution for σ_θ Along a Curved Beam a = 10 in, b = 12 in, t = .25 in, M = 300 lbf/in^2

When we approximate the beam as a straight beam, the analytical solution deviates slightly from the ANSYS solution.

Stress Distribution for σ_θ Along a Curved Beam a = 10 in, b = 12 in, t = .25 in, M = 300 lbf/in^2
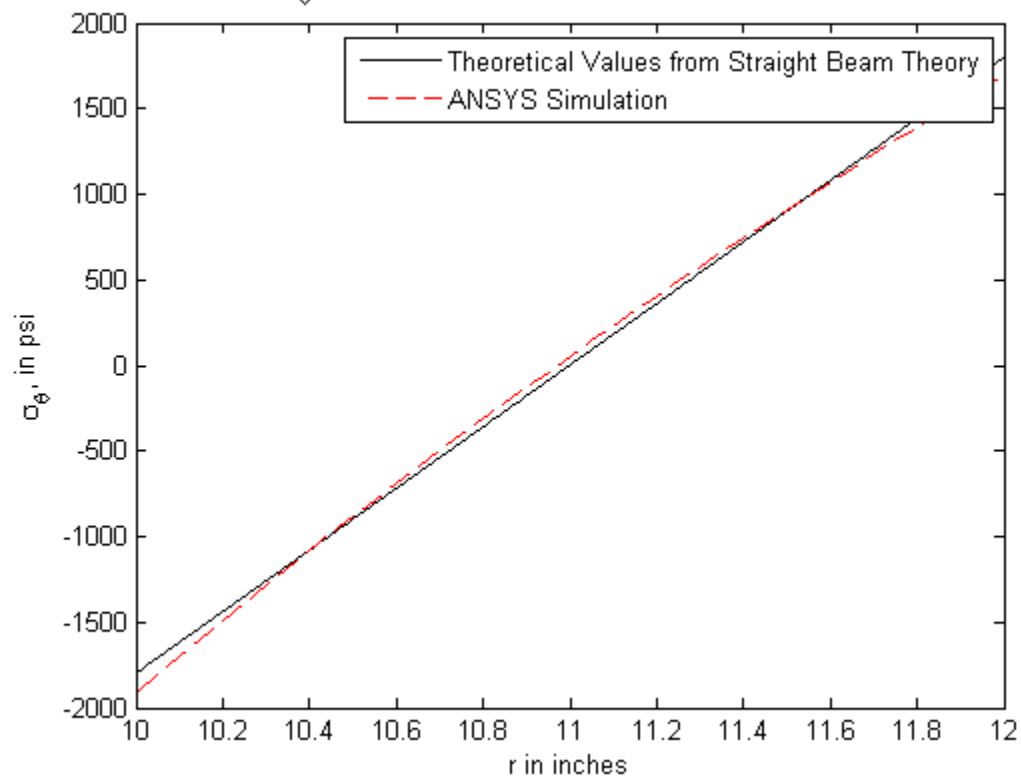
Now that we have gone through a simulation for bending of a curved beam, it is time to see if you can do the same on your own!

Go to Step 3 - Verification and Validation