Plate with a Hole (Results-Interpretation) - Pre-Analysis & Start-Up

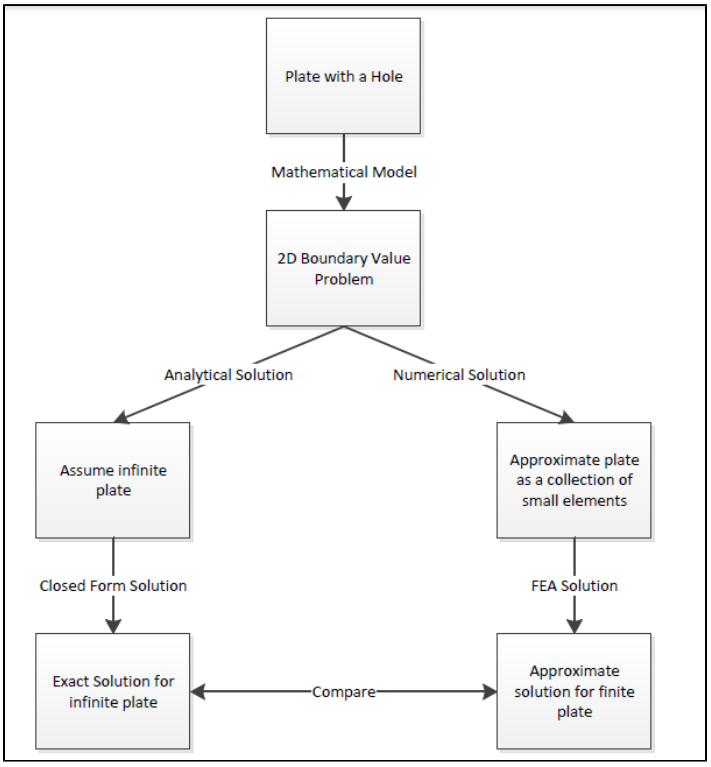
Author: Benjamin Mullen, Cornell University

Problem Specification
1. Pre-Analysis & Start-Up
2. Numerical Results
3. Verification and Validation
Exercises
Comments

Pre-Analysis and Start-Up

Analytical vs. Numerical Approaches

We can either assume the geometry as an infinite plate and solve the problem analytically, or approximate the geometry as a collection of "finite elements", and solve the problem numerically. The following flow chart compares the two approaches.



Let's first review the analytical results for the infinite plate. We'll then use these results to check the numerical solution from ANSYS.

Analytical Results

Displacement

Let's *estimate* the expected displacement of the right edge relative to the center of the hole. We can get a reasonable estimate by neglecting the hole and approximating the entire plate as being in uniaxial tension. Dividing the applied tensile stress by the Young's modulus gives the uniform strain in the x direction.

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E}$$

$$\epsilon_{xx} = \frac{\Delta L}{L}$$

$$\Delta L = \frac{\sigma_{xx} \times L}{E}$$

$$\sigma = 1 \times 10^6 \text{ psi}, \; L = 5 \text{ inches}, \;\; \text{and } E = 29 \times 10^6 \text{ psi}$$

$$\Delta L = .1724 \text{ inches}$$

Multiplying this by the half-width (5 in) gives the expected displacement of the right edge as ~ 0.1724 in. We'll check this against ANSYS.

Sigma-r

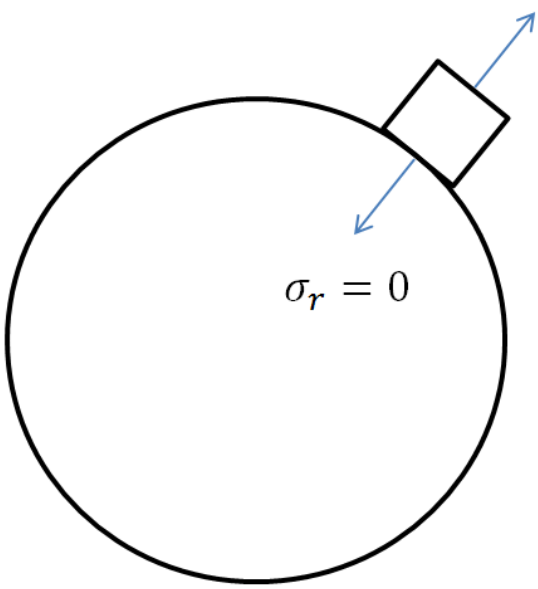
Let's consider the expected trends for Sigma-r, the radial stress, in the vicinity of the hole and far from the hole. The analytical solution for Sigma-r in an infinite plate is:

$$\sigma_r(r,\theta) = \frac{1}{2}\sigma_o[(1 - \frac{a^2}{r^2}) + (1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2})cos(2\theta)]$$

where *a* is the hole radius and Sigma-o is the applied uniform stress (denoted *P* in the problem specification). At the hole (r=a), this reduces to

$$\sigma_r = 0$$

This result can be understood by looking at a vanishingly small element at the hole as shown schematically below.



We see that Sigma-r at the hole is the normal stress at the hole. Since the hole is a free surface, this has to be zero.

For r>>a,

$$\sigma_r(r,\theta) = \sigma_r(\theta) = \frac{1}{2}\sigma_o[1 + cos(2\theta)]$$

Far from the hole, Sigma-r is a function of theta only. At theta = 0, Sigma-r ~ Sigma-o. This makes sense since r is aligned with x when theta = 0. At theta = 90 deg., Sigma-r ~ 0 which also makes sense since r is now aligned with y. We'll check these trends in the ANSYS results.

Sigma-theta

Let's next consider the expected trends for Sigma-theta, the circumferential stress, in the vicinity of the hole and far from the hole. The analytical solution for Sigma-theta in an infinite plate is:

$$\sigma_\theta(r,\theta) = \frac{1}{2}\sigma_o[(1+\frac{a^2}{r^2}) - (1+3\frac{a^4}{r^4})cos(2\theta)]$$

At r = a, this reduces to

$$\sigma_\theta = \frac{1}{2}\sigma_o(2 - 4cos(2\theta))$$

At theta = 90 deg., Sigma-theta = 3*Sigma-o for an infinite plate. This leads to a stress concentration factor of **3** for an infinite plate.

For r>>a,

$$\sigma_\theta(r,\theta) = \sigma_\theta(\theta) = \frac{1}{2}\sigma_o[1 - cos(2\theta)]$$

At theta = 0 and theta = 90 deg., we get

$$\sigma_\theta(0) = \frac{1}{2}\sigma_o[1 - cos(2(0))] = 0$$

$$\sigma_\theta(\frac{\pi}{2}) = \frac{1}{2}\sigma_o[1 - cos(2(\frac{\pi}{2}))] = \sigma_o$$

Far from the hole, Sigma-theta is a function of theta only but its variation is the opposite of Sigma-r (which is not surprising since r and theta are orthogonal coordinates; when r is aligned with x, theta is aligned with y and vice-versa). As one goes around the hole from theta = 0 to theta = 90 deg., Sigma-theta increases from 0 to Sigma-o. More trends to check in the ANSYS results!

Tau-r-theta

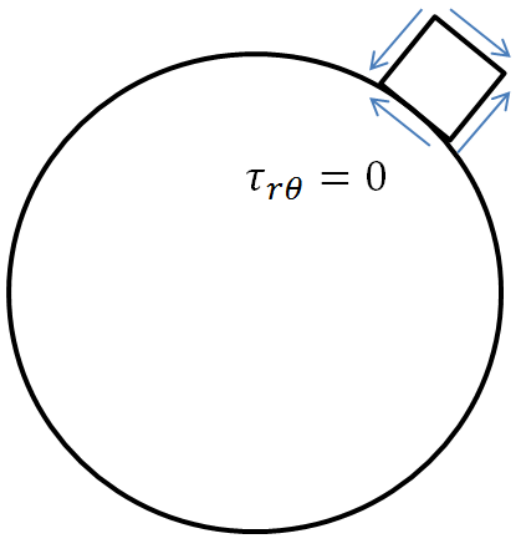
The analytical solution for the shear stress *Tau-r-theta* in an infinite plate is:

$$\tau_{r\theta} = -\frac{1}{2}\sigma_o(1 - 3\frac{a^4}{r^4} + 2\frac{a^2}{r^2})sin(2\theta)$$

At r=a,

$$\tau_{r\theta} = 0$$

By looking at a vanishingly small element at the hole , we see that Tau-r-theta at the hole is the shear stress at the hole. Since the hole is a free surface, this has to be zero.



For r>>a,

$$\tau_{r\theta}(r,\theta) = \tau_{r\theta}(\theta) = -\frac{1}{2}\sigma_o sin(2\theta)$$

We can deduce that, far from the hole, Tau-r-theta = 0 both at theta = 0 and theta = 90 deg. Even more trends to check in ANSYS!

Sigma-x

First, let's begin by finding the average stress, the nominal area stress, and the maximum stress with a concentration factor.

$$\sigma_o = \frac{F}{A} = \frac{1000000}{.2 \times 5}\frac{\text{lb}}{\text{in}^2} = 1 \times 10^6 \text{ psi}$$

$$\sigma_{nominal} = \frac{F}{A} = \frac{1000000}{.2 \times (5 - .5)}\frac{\text{lb}}{\text{in}^2} = 1.111 \times 10^6 \text{ psi}$$

The concentration factor for an infinite plate with a hole is K = 3. The maximum stress for an innite plate with a hole is

$$\sigma_{max} = K \times \sigma_o$$

$$\sigma_{max} = (3.0)(1.0 \times 10^6\text{psi}) = 3.0 \times 10^6\text{psi}$$

Although there is no analytical solution for a nite plate with a hole, there is empirical data available to find a concentration factor. Using a Concentration Factor Chart (3250 Students: See Figure 4.22 on page 158 in Deformable Bodies and Their Material Behavior), we find that d/w = 1 and thus K ~ 2:73
Now we can find the maximum stress using the nominal stress and the concentration factor

$$\sigma_{max} = K \times \sigma_{nominal} = (2.73)(1.111 \times 10^6 \text{ psi}) = 3.033 \times 10^6 \text{ psi}$$

Numerical Solution using ANSYS

You might be thinking: will we ever get to ANSYS? OK, let's load the numerical solution for the finite plate into ANSYS and see how it compares with the above analytical solution for the infinite plate. First, start by downloading the files here.

The zip file should contain the following:

- Plate With a Hole_files folder
- Plate With a Hole.wbpj

Please make sure to extract both of these files from the zip folder, the program will not work otherwise. (Note: The solution was created using ANSYS workbench 12.1 release, there may be compatibility issues when attempting to open with older versions).

2. Double click "Plate With a Hole.wbpj" - This should automatically open ANSYS workbench (you have to twiddle your thumbs a bit before it opens up). You will be presented with the ANSYS solution.

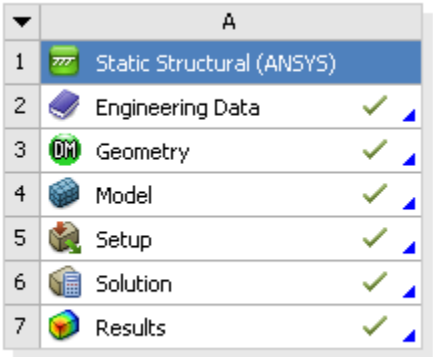
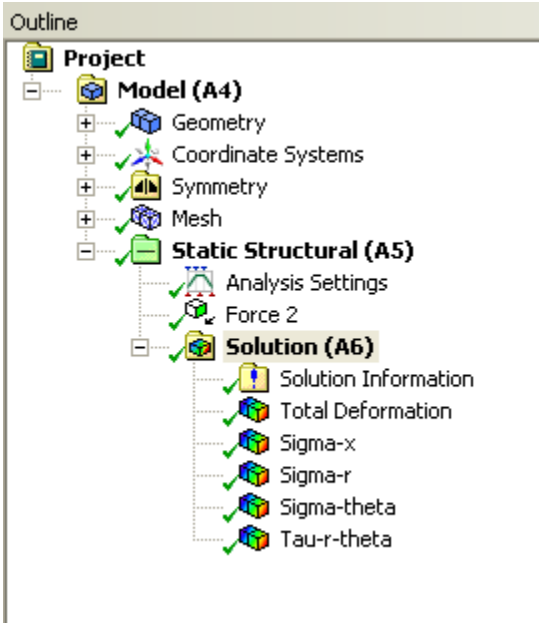


Plate With a Hole

A tick mark against each step indicates that that step has been completed.

3. To look at the results, double click on "Results" - This should bring up the ANSYS Mechanical application (again you have to twiddle your thumbs a bit before it opens up).

4. On the left-hand side there should be an "Outline" toolbar. Look for "Solution (A6)".



We'll investigate the items listed under Solution in the next step of this tutorial.

Go to Step 2: Numerical Results

Go to all ANSYS Learning Modules