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Flocculation Model and Collision Potential for Reactors with Flows Characterized by High Peclet Numbers

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Abstract

An analytical model is developed to characterize flocculation in the context of flow regimes with high Peclet numbers such as would occur in serpentine flow reactors. These flow conditions are obtained in gravity-driven hydraulic flocculators without mechanical agitation that are an important component of sustainable water treatment systems where energy efficiency and cost are receiving increasing emphasis. The model incorporates a fractal description of flocs, estimates of floc separation distances, estimates of relative velocities of floc particles dependent on the relevant flow regime, and provides an overall prediction of the required reaction time for formation of settleable flocs based on influent turbidity, alum dose, and energy dissipation rate. Viscosity is shown to be significant for the early stage of flocculation and turbulent eddies are shown to be significant for the final stage of flocculation. The collision potential defined as the product of the hydraulic residence time ( ) and the cube root of the energy dissipation rate ( ), i.e., , is shown to be a better predictor of flocculator performance than the conventional product of and the velocity gradient (G), i.e., G .

Keywords : hydraulic flocculator, analytical model, fractal dimension, floc density, sedimentation, floc volume fraction, collision potential, AguaClara

Introduction

Hydraulic flocculators are devices in which laminar or turbulent fluid motions originating from reactor geometry cause collisions of flocculent particles. Hydraulic flocculators can be built to operate with only a few centimeters of potential energy. The ability of hydraulic flocculators to function without any moving parts or external power sources makes them far more energy efficient and sustainable than mechanical flocculators as a technology for surface water treatment.

Existing design guidelines suggest that the Camp velocity gradient (Camp, 1955) , G , be varied from 100 s^{-1} to 10 s^{-1} (Schulz et al., 1984) in a flocculator to produce settleable flocs. However, the use of the Camp velocity gradient as a flocculation reactor design parameter has been criticized because the important turbulent eddies causing flocculation are larger than the Kolmogorov microscale (Cleasby, 1984) . In addition, there has been a continuing search to provide a fundamental basis for the range of recommended design values for the product of velocity gradient and hydraulic residence time ( ) in flocculators. Schultz and Okun (1984) suggested a G range from 20,000 to 150,000. Lawler and Nason (2004) have suggested the use of the dimensionless product G_{Floc} , where $_{Floc}$ is the floc volume fraction, for design of mechanically mixed flocculators. However, in hydraulic flocculators $_{Floc}$ is not a constant and thus this formulation is not well defined for hydraulic flocculators. Because the use of the velocity gradient in the design of turbulent flow flocculators is suspect, a flocculation model that also incorporates turbulent flow transport processes is needed.

The need for fundamentally based design guidelines prompted us to model the influence of shear, turbulence, and residence time on the performance of the flocculation process. The flocculation process is difficult to model because the floc sizes evolve over a scale ranging from m to mm. The flow field at the particle level can be laminar or turbulent. In addition, floc density and the floc volume fraction are known to vary with floc size. These complications have precluded the development of simple analytical models that describe the entire flocculation process.

Hydraulic flocculators with their serpentine flow path may be idealized as arbitrary flow reactors with high Peclet numbers and thus it is reasonable to simplify the problem by assuming that floc size increases as particles travel along the flow path, and to assume that only a narrow floc size distribution exists at each location in a hydraulic flocculator. In a simplified model each primary particle collides with one other primary particle to form a doublet. Doublets then collide to form flocs with 4 primary particles and this process continues over the length of the flocculator. This simplifying assumption with respect to floc growth serves as the basis for the hydraulic flocculator model developed here. Knowledge of the time required for each of the collisions in the geometric series permits calculation of the total time required for flocculation.

Floc Relative Velocities

Floc collisions are generated by differential velocities. Three mechanisms contribute to the differential velocities, Brownian motion, velocity gradients arising from laminar and turbulent fluid motion, and differential sedimentation. Brownian motion is an effective transport mechanism for particles that are smaller than 1 m (Gregory, 1998) . Thus, Brownian motion plays an important role in the rapid mix phase as it transports the coagulant to the colloids. For the modeling effort described herein we begin the analysis after the rapid mix and assume that Brownian motion is insignificant in comparison with the transport caused by larger scale fluid motions. Flocculators with high Peclet numbers (advection large compared with dispersion) are expected to generate a relatively narrow range of floc sizes at any location within the flocculator precluding significant collisions by differential sedimentation. Thus, floc collisions in hydraulic flocculators are expected to be primarily caused by fluid velocity gradients at both the viscous and turbulent scales.

An estimate of the relative velocities of flocs is required for the analysis of the average time required for each collision. The relative velocity of two flocs is a function of their separation distance. In the inertial range, flocs are given differential velocities by eddies that are similar in size to the separation distance of the flocs. Larger eddies tend to have larger velocities, thus flocs that are farther apart tend to have higher relative velocities. Flocs that are separated by distances that are within the viscous subrange have differential velocities caused by velocity gradients and thus their relative velocities also increase with separation distance.

The average separation distance between flocs increases as the flocs grow in size. A measure of the average floc separation distance can be obtained from the floc volume fraction, $_{Floc}$.

(1)

where $\forall_{Occupied}$ is the average suspension volume occupied by each floc, equal to the total suspension volume divided by the number of flocs, $\forall_{Floc}$ is the floc volume, C_{Floc} is the concentration of the floc solids and $\square_{Floc}$ is the density of the floc.

Substituting the equation for the volume of sphere as a measure of the volume occupied by the floc and the separation distance cubed as a measure of the occupied suspension volume gives:

$$\phi_{Floc} = \frac{\frac{\pi d^3}{6}}{L_{Separation}^3} \quad (2)$$

Wh *ere* $L_{Separation}$ is an estimate of average distance between flocs, and *d* is the diameter of the floc particle. Rearranging equation 2 provides an estimate of the floc separation distance.

$$L_{Separation} = d\left(\frac{\pi}{6\phi_{Floc}}\right)^{\frac{1}{3}} \quad (3)$$

The floc volume fraction, ϕ_{Floc} , is related to the volume fraction of the initial or primary particles (ϕ_{Floc_0}) , the diameter of primary particles having a fractal dimension of 3 that combine to form flocs (d_o), and the fractal dimension ($D_{Fractal}$ --------) as follows (Weber-Shirk et al., 2009) :

$$\phi_{Floc} = \phi_{Floc_0}\left(\frac{d}{d_0}\right)^{3-D_{Fractal}} \quad (4)$$

where ϕ_{Floc_0} is based on the composition of the initial suspension which is assumed here to consist of colloidal clay particles and precipitated coagulant, $Al(OH)_3$ (s). The derivation of equation 4 is included in the supplemental information to this paper.

$$\phi_{Floc_0} = \frac{C_{Al(OH)_3}}{\rho_{Al(OH)_3}} + \frac{C_{Clay}}{\rho_{Clay}} \quad (4)$$

Substituting equation 4 into equation 3 shows the relationship between fractal dimension and the floc separation distance.

$$L_{Separation} = d_0\left(\frac{d}{d_0}\right)^{\frac{D_{Fractal}}{3}}\left(\frac{\pi}{6\phi_{Floc_0}}\right)^{\frac{1}{3}} \quad (5)$$

The separation distance does not grow in proportion to the floc diameter when the fractal dimension is less than 3. The separation distance is used below to determine if the dominant transport mechanism for flocculation is viscous shear or turbulent eddies.

The Kolmogorov length scale (η_K) sets the boundary between the viscous range where turbulent eddies do not penetrate and the inertial range where viscosity is insignificant (Tennekes et al., 1972) .

$$\eta_K = \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}} \quad (6)$$

where: ν is kinematic viscosity, and ε is the energy dissipation rate.

Previous researchers have compared the floc diameter to the Kolmogorov length scale (equation 6) to determine if the transport process for floc collisions was turbulent eddies or viscous shear (Bouyer et al., 2005; Cleasby, 1984) . The floc diameter is an appropriate length scale to assess the role of turbulence on floc breakup, but for floc collisions the important length scale governing transport is the average distance between flocs. It is over this separation distance that either inertia dominated eddies or viscous dominated velocity gradients will act to transport a floc to cause a collision. As shown below, one of the results of an analysis based on floc separation distance rather than on floc diameter is that more of the flocculation regime occurs in the inertial range because the length scale that is compared with the Kolmogorov scale is larger.

If the average floc separation distance is less than the Kolmogorov length scale, then the relative floc motions are governed by viscosity-dominated shear. Flocculation has been frequently described by the Camp velocity gradient (Camp, 1955; Han et al., 1992; Lawler et al., 2004) which implicitly assumes that the interactions between flocs are dominated by viscous shear. However, it has been known for some time that large floc interactions are governed by turbulence and that the Camp velocity gradient does not capture the physics of the particle interactions (Cleasby, 1984) . Given the possible range of separation distances between flocs, we derive below equations for relative velocities between flocs based on both viscous and turbulent scales.

For the viscous scale the relative velocity, $V_{r_{viscous}}$, is a function of the energy dissipation rate ( ) , kinematic viscosity ( ), and the distance between the two flocs, ($L_{Separation}$).

$$V_{r_{viscous}} = f(\varepsilon, \nu, L_{Separation}) \quad (7)$$

Assuming a linear velocity gradient requires that the relative velocity be proportional to $L_{Separation}$ and thus the functional relationship can be rewritten as:

$$V_{r_{viscous}} = L_{Separation} f(\varepsilon, \nu) \quad (8)$$

Dimensional equality requires that the energy dissipation rate and kinematic viscosity be combined in a fashion that results in units of inverse time, giving:

$$V_{r_{viscous}} \propto L_{Separation} \sqrt{\frac{\varepsilon}{\nu}} \quad (9)$$

This formulation for the relative velocities between two flocs reveals the significance of the velocity gradient, G , for the viscous subrange, where (Cleasby, 1984) :

$$G \propto \sqrt{\frac{\varepsilon}{\nu}} \quad (10)$$

At separation distances that exceed the Kolmogorov length scale, viscosity is no longer important and thus the relative velocity in the inertial range, $V_{r_{inertial}}$, only can be a function of the energy dissipation rate and the separation distance.

$$V_{r_{inertial}} \propto f\left(\varepsilon, L_{Separation}\right) \quad (11)$$

Given this two parameter dependency, dimensional equality can only be satisfied if (Tennekes et al., 1972) :

$$V_{r_{inertial}} \propto \left(\varepsilon L_{Separation}\right)^{1/3} \quad (12)$$

The relative velocities between flocs are thus shown to have different relationships depending on whether the separation distance is greater or less than the Kolmogorov length scale.

Collision Times

Development of a model that can characterize the time required for collisions between floc particles can provide guidance for the hydraulic residence time needed in flocculator design. Flocs that have yet to collide have an average separation distance given by equation 5 and a relative velocity given by either equation 9 or 12 . The collision process may include many close encounters, separation distances that vary widely, and uncertainty as to which two flocs are actually going to collide. The duration of the collision process should, on average, be determined by the time required for a floc to move through a volume of water equal to the suspension volume occupied by the floc. The average volume of the suspension that must be “cleared” for a collision to occur is the total suspension volume divided by the number of flocs.

Since flocs are very porous, it is assumed that fluid streamlines pass through the flocs (Adler, 1981) an d thus a rectilinear model (Thomas et al., 1999) for particle collisions is reasonable. As a result, the suspension volume through which a floc has traveled in its path to a collision (i.e., the volume cleared, $\forall_{Cleared}$) is proportional to the projected area swept by the floc, the relative velocity of two flocs (V_r), and time. In a rectilinear model for the collision between two equal diameter flocs a collision will occur if the center to center distance between flocs is equal to the floc diameter. Thus, the diameter of the swept area is equal to twice the diameter of the floc. These factors give the following relationship.

$$\forall_{Cleared} \propto d^2 V_r t \quad (13)$$

The suspension volume occupied by a floc is given by the volume of a floc divided by the floc volume fraction.

$$\forall_{Occupied} = \frac{\frac{\pi}{6} d^3}{\phi_{floc}} \quad (14)$$

The distance that two flocs travel relative to each other before a collision can be estimated by taking the volume occupied by a floc and dividing it by the area that a floc clears. Setting the clearance volume (equation 13) equal to the suspension volume occupied by a floc (equation 14) means that a floc will have cleared a volume of water equal to the suspension volume occupied and thus it will, on average, have traveled the distance needed to collide with another floc so that time, t , becomes the time required for a collision, t_c . .

$$t_c = \frac{d}{6 \phi_{floc} V_r} \quad (15)$$

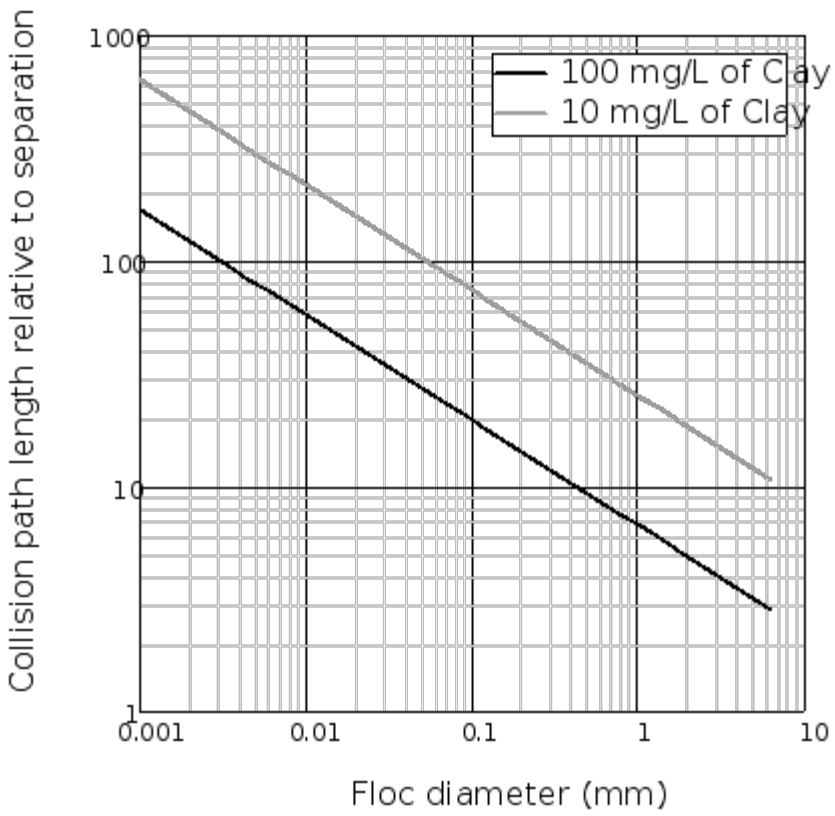
The distance that two flocs travel relative to each other before collision divided by the average floc separation distance provides a sense of the importance of the path traveled prior to floc collisions. The distance traveled before collision is equal to the relative velocity times the collision time from equation 15 . This can then be normalized by the average separation distance between flocs using equation 5 . Substitution for ϕ_{floc} using equation 4 gives:

$$\frac{L_{Collision}}{L_{Separation}} \approx \frac{1}{6}\left(\frac{6}{\pi}\right)^{\frac{1}{3}} \alpha_{Floc_0}^{\frac{2}{3}} \left(\frac{d}{d_0}\right)^{\frac{2D_{Fractal}}{3}-2} \quad (16)$$

A plot of equation 16 for two different clay concentrations is shown in Figure 1. To create a reasonable alum dose the following empirical equation was used (Hurst et al., 2009) . Experimental data on which this relationship is based are included in the supplemental information to this paper.

$$C_{Alum} \approx 7.8\left(\frac{mg}{L}\right)\left(C_{Clay}\left(\frac{L}{mg}\right)\right)^{0.4} \quad (17)$$

The analysis reveals that distance traveled prior to collision is many times larger than the average distance between flocs except for the case of high turbidity and large flocs.


Collision path length relative to separation
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100
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Floc diameter (mm)
100 mg/L of Clay
10 mg/L of Clay

Distance travelled by flocs prior to collision relative to their average separati

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Figure 1. .

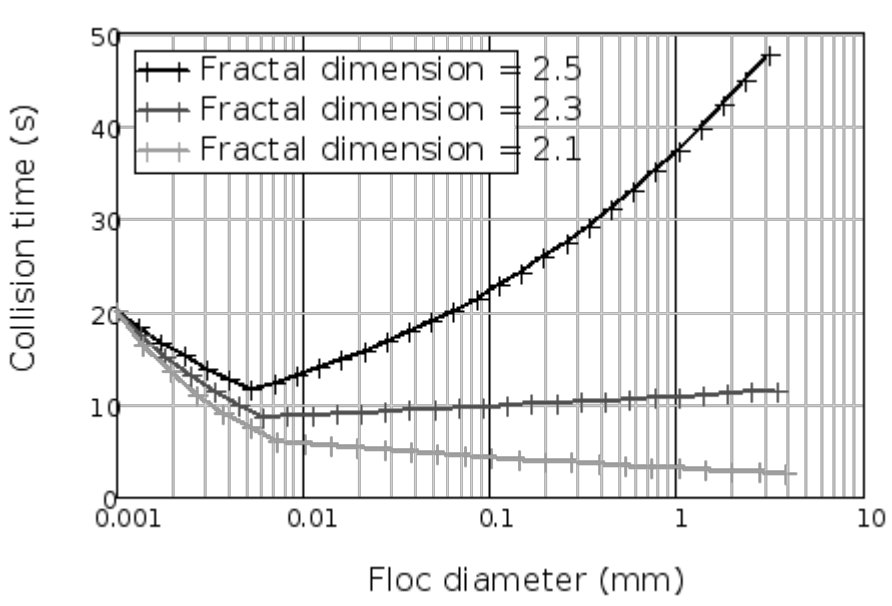
Note that the collision time is dependent on the floc volume fraction and the relative velocity of the floc and its closest neighbor and that both of those terms can be expressed as functions of the floc diameter. Substituting equations 4 , 5 , and 9 into equation 15 results in the collision time for flocs with separation distances less than the Kolmogorov scale.

$$t_{c_{viscous}} : \frac{1}{6}\left(6\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\right)_{Floc_0}^{\frac{2}{3}}\left(\frac{d}{d_0}\right)^{\frac{2D_{Fractal}}{3}-2} \quad (18)$$

For fractal dimensions less than 3 the collision time decreases as the floc size increases. Equation 18 also applies to laminar flow flocculators over the entire range of floc sizes and separation distances since laminar flow flocculators don’t have turbulent eddies at any length scale. The companion equation for the collision time for separation distances greater than the Kolmogorov scale is obtained by substituting equations 4 , 5 , and 12 into equation 15 :

$$t_{c_{inertial}} : \frac{1}{6}\left(\frac{6}{\pi}\right)^{\frac{1}{3}} \varepsilon^{-\frac{1}{3}} d^{\frac{2}{3}} \nu_{Floc_0}^{\frac{8}{9}} \left(\frac{d}{d_0}\right)^{\frac{8-D_{Fractal}-3}{9}} \quad (19)$$

Equations 18 and 19 were evaluated over a range of floc sizes and the model results are shown in Figur e 2 . The model curves illustrate that the collision times predicted by the model are greater than would have been predicted if the model had been based exclusively on laminar or turbulent transport for collisions because laminar based collision durations monotonically decrease as the floc size increases and turbulent based collisions continue to be faster than laminar based collisions as the floc size decreases. The collision time is independent of floc diameter in the inertial range when the fractal dimension is equal to 9/4 or 2.25 (see the exponents of *d* in equation 19). The predicted collision times are remarkably constant throughout the flocculation process for the fractal dimension of 2.3.



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Collision Potential

Floc collisions require time and relative velocities. Traditionally the (Camp, 1955) velocity gradient was multiplied by reactor residence time to characterize the required collision potential. Equations for the required collision potential for both laminar flow and turbulent flow flocculators are developed below and the results are compared with traditional flocculator design guidelines.

Assuming sequential collisions occur between like sized particles as flocs grow in a hydraulic flocculator, the diameter of a fractal floc after j collisions is (Weber-Shirk et al., 2009; also, see supplemental information) :

$$d \approx d_0 j^{\frac{1}{D_{Fractal}}} \approx d_0 2^{\frac{j}{D_{Fractal}}} \quad (20)$$

where $i = 2^j$ and i is the number of primary particles in the floc. The total number of sequential collisions required to grow a floc from 1 m to 3 mm can be obtained by solving equation 20 for j. Approximately 24 collisions are required for a fractal dimension of 2.1 and 27 collisions for a fractal dimension of 2.3. From Figure 2 it can be seen that the total flocculation time can be obtained by summing the individual collision times. The results for total collision time as a function of the energy dissipation rate are shown in Figure 3.

Typical hydraulic flocculators are designed to have a G value of 10 to 100 s^{-1} and a residence time of 900 to 1800 seconds (Schulz et al., 1984) . Assuming a kinematic viscosity of 1 mm^2/s results in an energy dissipation rate of 0.1 to 10 mW/kg. The zone of time and energy dissipation corresponding to conventional design is shown in Figure 3 and is similar to that predicted to be needed by the model for turbidities $\geq$ 1 NTU.

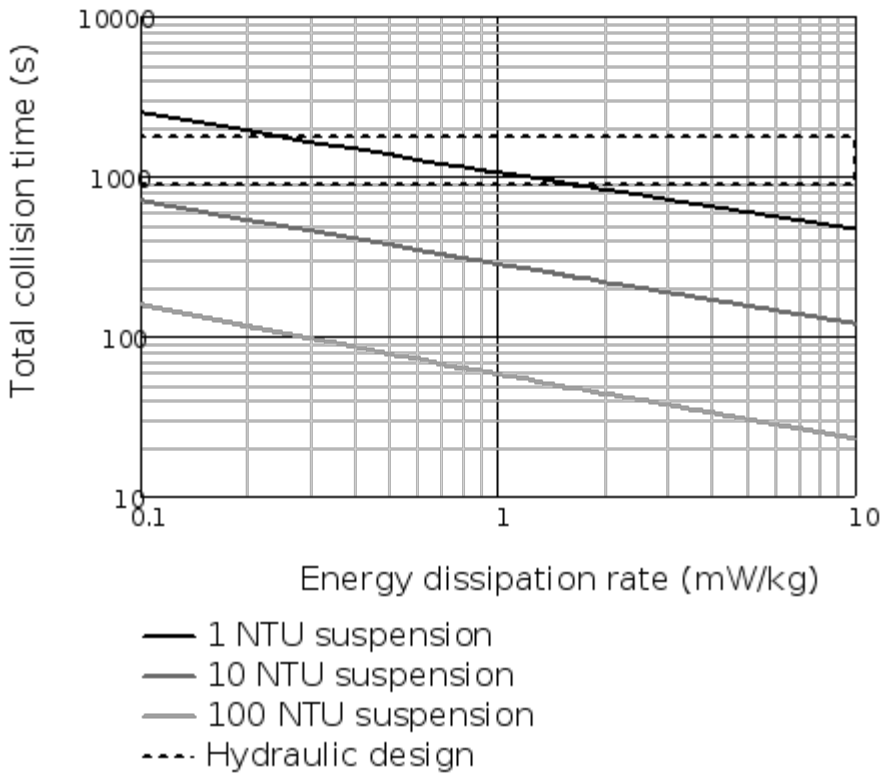
Total collision time (s)
10000
1000
100
10
0.1
1
10
Energy dissipation rate (mW/kg)
1 NTU suspension
10 NTU suspension
100 NTU suspension
Hydraulic design

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Figure 3. .

Equation 18 (with the floc diameter, d , replaced with the collision number, j , using equation 20) can be summed up for all of the floc collisions in a laminar flow flocculation process.

$$\sum_{j=0}^{n-1} t_{c_n} = \frac{1}{6}\left(\frac{6}{\pi}\right)^{\frac{1}{3}}\left(\frac{\rho}{\mu}\right)^{\frac{1}{2}} d_{Floc_0}^{\frac{2}{3}} \sum_{j=0}^{n-1} 2^{j\left(\frac{2}{3}-\frac{2}{D_{Fractal}}\right)} \quad (21)$$

The sum of all of the collision times is equal to the required total flocculation residence time, . The total number of collisions, n , can be obtained by rearranging equation 20 and replacing j with n.

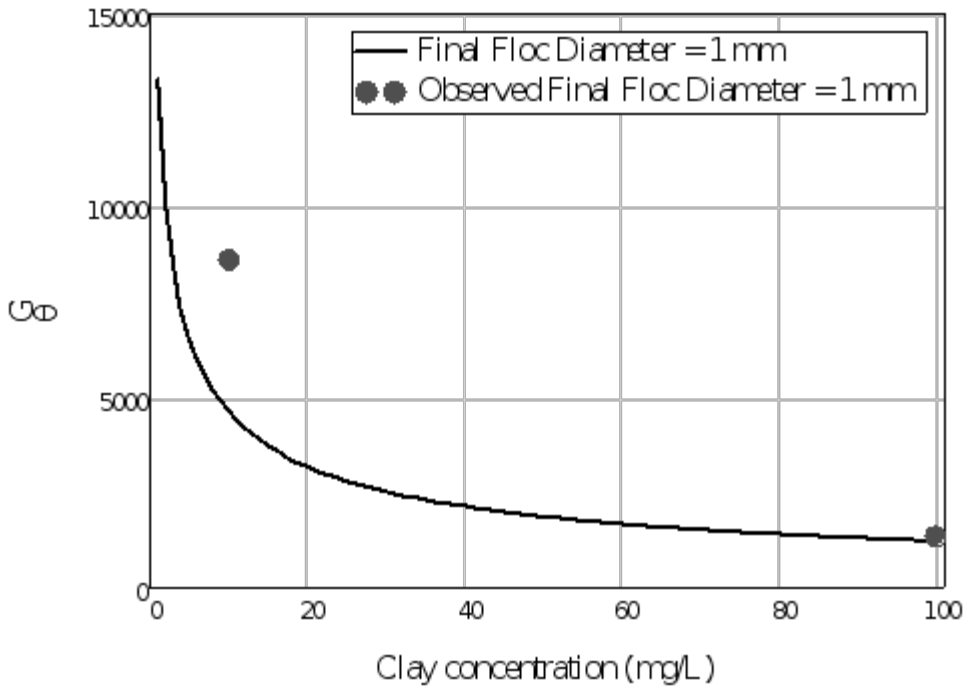
$$n = D_{Fractal} \log_2\left(\frac{d_n}{d_0}\right) \quad (22)$$

The diameter of the final floc, d_n , is produced after n collisions. Equation 21 can be compared with the conventional flocculation parameter G by replacing $\left(\frac{\rho}{\mu}\right)^{\frac{1}{2}}$ with the velocity gradient G to obtain:

$$G = \frac{1}{6}\left(\frac{6}{\pi}\right)^{\frac{1}{3}} d_{Floc_0}^{\frac{2}{3}} \sum_{j=0}^{n-1} 2^{j\left(\frac{2}{3}-\frac{2}{D_{Fractal}}\right)} \quad (23)$$

Equation 23 is comparable to the formulation given by Ives in the context of sludge blanket flocculators in which he proposed that $G\overline{\overline{\Pi}}$ was a constant. (Ives, 1968) However, the exponent on the floc volume fraction is reduced to 2/3 because both the relative velocities (equation 9) and the separation distance (equation 5) between flocs are affected by the floc volume fraction. For a fractal dimension of 2.3 and for 27 sequential collisions the parameter $\frac{1}{6}\left(\frac{6}{\pi}\right)^{\frac{1}{3}}\sum_{j=0}^{n-1}2^{\left(\frac{2}{3}-\frac{2}{D_{Fractal}}\right)j}$ has a value of 1.5 and thus $G\overline{\overline{\Pi}}_{Floc_0}^{\frac{2}{3}}$ is expected to have a value of at least 1.5 for laminar flow flocculation. Although the floc volume fraction is not a constant in high Peclet number or batch flocculators, the required value of $G\overline{\overline{\Pi}}_{Floc_0}^{\frac{2}{3}}$ explicitly accounts for the changing floc volume fraction as the flocs grow in size and become more porous. Equation 23 is plotted in Figure 4 as a function of the clay concentration for two different final floc diameters. The required flocculation potential characterized by G is largest for low turbidity waters because the separation distance between flocs is higher and thus collision times are longer. The difficulty of flocculating low turbidity water is well known and the model results suggest that flocculators must be designed for the low turbidity constraint. However, it is possible that the need for flocculators to produce low residual turbidity after sedimentation is an even more severe constraint for high turbidity waters.

Two estimates of the residence time where the floc diameter reached 1 mm were obtained based on observation in a transparent tube flocculator that was operated at 10 and 100 mg/L kaolin clay with alum dosages set according to equation 17 . The location in the tube flocculator was then converted to G based on the hydraulic residence time and the velocity gradient in the tube flocculator. The estimates are shown on Figure 4 and agree well at 100 mg/L, but suggest that the model predictions may be low for the 10 mg /L clay suspension.


15000
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60
80
100
Final Floc Diameter = 1 mm
Observed Final Floc Diameter = 1 mm
Clay concentration (mg/L)

Total flocculation potential characterized by G (equation 23) as a function of

the clay concentration. Note that this graph is only applicable to laminar

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collision. The fractal dimension was 2.3. The two measured values were

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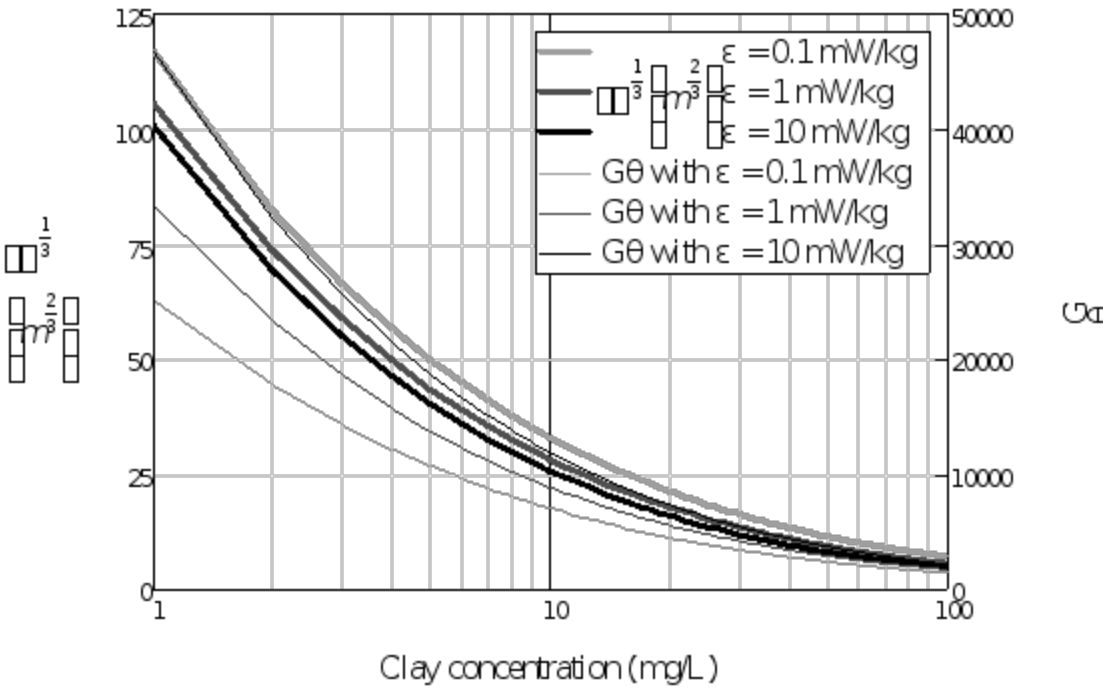
Figure 4.).

Application of the flocculation model to full-scale turbulent flow hydraulic reactors requires inclusion of both inertial and viscous mediated collisions. For design purposes it would be useful to determine if the required collision potential is better characterized by the dimensionless parameter G as has been done historically, or since t_c is proportional to $\varepsilon^{-\frac{1}{3}}$ in equation 19 if a parameter based on $\varepsilon^{\frac{1}{3}}$ would be more appropriate. If the collision time is dominated by viscous transport, then G (in which G is proportional to $\varepsilon^{\frac{1}{2}}$) would be the correct parameter to describe the flow. However, if the collision time is dominated by eddies, then $\varepsilon^{\frac{1}{3}}$ will be the best parameter. Note that inclusion of viscosity in the Camp velocity gradient ($G = \sqrt{\frac{\varepsilon}{\nu}}$) renders $G\theta$ as a dimensionless design parameter for viscous flow. Examination of equation 20 shows that inclusion of $d_0^{2/3}$ would be appropriate to create a dimensionless parameter (i.e. $\varepsilon^{\frac{1}{3}} d_0^{2/3}$) for design of flocculators where flow is turbulent. However, we prefer to keep suspension properties, that determine the collision potential required to form flocs of a given size, separate from reactor properties that determine the collision potential provided by the reactor. The energy dissipation rate and the reactor residence time are reactor properties that can be designed whereas the primary particle diameter is a suspension property. Thus $\varepsilon^{\frac{1}{3}}$ is an appropriate measure of the collision potential provided by a reactor with a uniform energy dissipation rate.

For comparison purposes, the total collision times for the sequential collisions required to create a 3 mm diameter floc as a function of clay concentration were calculated and then were multiplied by either to the 1/3 or G and the results are shown in Figure 5. The collision times were based on equation 18 for floc separation distances less than the Kolmogorov length scale and were based on equation 19 for floc separation distances larger than the Kolmogorov length scale. The times for the 27 collisions were summed to obtain an overall flocculation time. Ideally the plots for the three energy dissipation rates would converge to a single curve for the correct parameterization. However, the flocculation process includes both viscous dominated and inertial dominated reactions and thus no single parameter can completely describe the process.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The model results show that at any given clay concentration the variability in the collision potential expressed as $\left(\frac{1}{3}\right)$ is much less than the variability in the parameter G Flocculator design is governed by the need to be able to flocculate low turbidity suspensions. Thus, the predicted required collision potential for low turbidity suspensions is most significant. For low turbidity suspensions collisions are dominated by inertia because flocs are spaced further apart and the separation distances are larger than the Kolmogorov length scale for more of the flocculation process. For turbulent flow hydraulic flocculators the collision potential varies with $\left(\frac{1}{3}\right)$ because the majority of the collisions time is dominated by inertial transport rather than by viscous transport.

Variability in collision potential, as expressed by the maximum collision potential normalized by the minimum over the range of energy dissipation rates from 0.1 to 10 mW/kg and evaluated at 1 mg/L of clay, is 1.86 for G and 1.16 for $\overline{\varepsilon}^{1/3}$. Thus $\overline{\varepsilon}^{1/3}$ is significantly better as a design parameter than G. The 1/3 exponent for the energy dissipation rate is consistent with the analysis by Bouyer et al. (2005). The fractal flocculation model results suggest that a collision potential $\overline{\varepsilon}^{1/3}$ of 100 m $^{2/3}$ (or $\overline{\varepsilon}^{1/3} C_0^{2/3}$ = 10 ^{6} for a typical primary particle size of 1 μm) would be a minimum design guideline for hydraulic flocculators that need to be able to treat low turbidity inputs. The collision potential only is a function of length and thus is expected to be a function of reactor geometry and possibly other dimensionless parameters such as the Reynolds number.

The flocculation model presented here describes mean floc size as a function of reaction time. It does not describe the floc size distribution and thus can not be used to predict the residual turbidity after sedimentation. Given that the function of flocculators is to produce suspensions with a low residual turbidity after sedimentation it is anticipated that additional reaction time (or collision potential) is desirable to allow large flocs to continue to clear the suspension of small flocs with low terminal velocities.

The model is based on an assumed uniform energy dissipation rate and thus the predicted values of the required collision potential apply to reactors with uniform energy dissipation rates. Although it is possible to design a hydraulic flocculator using a reasonable value of $\overline{\varepsilon}^{1/3}$ it is worth noting that ideally $\overline{\varepsilon}^{1/3}$ would not be based on an averaged value of $\overline{\varepsilon}^{1/3}$ calculated from the total energy dissipation in the reactor, but rather on a flow weighted summation of $\overline{\varepsilon}^{1/3}$ over the entire reactor volume. Characterization of the relationship between reactor geometry, Reynolds number, and $\overline{\varepsilon}^{1/3}$ is a component of a hydraulic flocculator reactor model that is the subject of ongoing research.

Conclusions

The fractal flocculation model makes reasonable predictions of flocculation collision potential requirements that are similar to recommended empirical design guidelines. Model calculations show that the majority of the hydraulic flocculation process for low turbidity suspensions is governed by inertia-dominated collisions of flocs. Therefore, the collision potential characterized by $\square\square^{\frac{1}{3}}$ is the preferred design parameter for high Peclet number and batch, turbulent flow flocculators instead of *G*

Laminar flow tube flocculators can be characterized by *G* The parameter $G\square\square_{Floc_0}^{\frac{2}{3}}$ is predicted to have a value of approximately 1.5 for the creation of 3 mm diameter flocs.

Low turbidity suspensions require large collision potentials and thus hydraulic flocculators must be designed to flocculate the lowest turbidity suspension that must be treated.

The reader is cautioned that the flocculation model is constrained by several simplifying assumptions. For example, the flocculation model assumes an idealized hydraulic flocculator in which sequential collision of same sized flocs occurs. Also, the model does not consider floc break up. The relative velocities between flocs are estimated and experimental determination of constants of proportionality in these constitutive equations is needed. The effects of streamline divergence around flocs has been neglected. Presumably the magnitude of the required correction would be a function of floc porosity and hence of floc diameter.

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