Validation of Turbulence Models:

Turbulence Modeling Resolution Problem

Computational Fluid Dynamics works by iteratively changing the values of the variables to reduce the residuals, or errors of the governing equation. The governing equations in our model are conservation of mass and conservation of momentum as shown below in Eq. 1 and Eq. 2:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

Equation 1. Conservation of Mass

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho g_x
\]

Equation 2. Conservation of Momentum

The conservation of momentum equation can be simplified to Reynolds Averaged Navier Stokes Equation (for x,y directions) in Fig. 3 by estimating velocity and pressure in terms of mean value (\( u \)) and fluctuation (\( u' \)) and averaging all terms as shown below:

\[
\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \bar{u}_i = \bar{f}_i - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \left( \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} - \frac{\partial u'_i u'_j}{\partial x_j} \right)
\]

Equation 3. Reynolds Averaged Navier Stokes Equation

With the conservation of mass equation, and conservation of momentum equations in the x and y directions, there are three governing equations. However with the introduction of the velocity fluctuation variable (\( u', v' \)), as shown above there are four variables (\( u, v, u', v' \), p). Thus, the problem becomes unsolvable unless additional equations are formulated to relate the variables. This is the Turbulence Modeling Resolution Problem.

The k-epsilon model overcomes this problem by relating the product of the velocity fluctuation terms to gradients of the mean velocity.

\[
-u'_i u'_j = \nu_t \left( \frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j} \right)
\]

The \( \nu_t \) represents a variable which can be related to the turbulent kinetic energy (k) and the energy dissipation rate (\( \epsilon \)).
The turbulent kinetic energy ($k$) and energy dissipate rate ($\varepsilon$) are governed by the following equations.

For turbulent kinetic energy $k$

$$
\frac{\partial}{\partial t} (\rho k) + \frac{\partial}{\partial x_i} (\rho k u_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + P_k + P_b - \rho \varepsilon - Y_M + S_k
$$

For dissipation $\varepsilon$

$$
\frac{\partial}{\partial t} (\rho \varepsilon) + \frac{\partial}{\partial x_i} (\rho \varepsilon u_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{1\varepsilon} \frac{\varepsilon}{k} (P_k + C_{3\varepsilon} P_b) - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} + S_\varepsilon
$$

Thus, these definitions of $k$ and $\varepsilon$ enable the system to be fully resolvable with 6 variables ($u, v, p, v_t, k, \varepsilon$) and 6 equations.

The $k$- model is one approach to create a resolvable problem and many other methods have been formulated. Each model may be geared best to solving a particular type of problem, therefore it is necessary to gauge the performance of the turbulence model.

### Comparing Turbulence Models Using Back-step Example

To determine on the turbulence model to use, a flow over backstep was compared with the literature experimental data. Figure 1 shows the flow of $Re = 48000$ over the channel. In the middle of the channel, the flow separate due to the small step size of height $h$. The flow reattaches at about 7 times the step height further downstream. This flow properties is similar to the 180 degree bend in the flocculation tank where we have flow separation and reattachment downstream (Figure 2).

![Figure 1: Flow over backstep in a open channel (Re = 48000, Reattachment length = 7h)](image_url)
The back step flow was analyzed using K-, K- SST, K- realizable, K- RNG, RSM turbulence models. The reattachment points of all the turbulence models were determined so that the reattachment ratio could be compared with the experimental data.

Plotting the derivative $\frac{du}{dy}$, the change in direction of velocity in x direction with respect to y at the wall, the reattachment point is easily identified. At the wall, separated flow will give a negative $\frac{du}{dy}$, while reattaches flow has a positive $\frac{du}{dy}$ value. Figure 3 shows the derivative of $\frac{du}{dy}$ vs x direction for different turbulence models.

**Figure 2: Flow over 180 degree turn in flocculation tank**

**Figure 3: $\frac{du}{dy}$ for Different Turbulence Models**
Table 1 shows the comparison results of different turbulence models.

**Table 1: Reattachment ratio with different turbulence models**

<table>
<thead>
<tr>
<th>Turbulence Model</th>
<th>K-e</th>
<th>K-W SST</th>
<th>K-e realizable</th>
<th>RSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reattachment Ratio</td>
<td>0.195/0.038 = 5.13</td>
<td>0.242/0.038 = 6.37</td>
<td>0.235/0.038 = 6.18</td>
<td>0.2/0.038 = 5.26</td>
</tr>
</tbody>
</table>

From table 1, the K- model under-predicts the reattachment length, as known by most literature. K- SST and K- realizable gives the most accurate representation of the back step flow with reattachment length of 6.37 and 6.18. However, from literature reviews, K- realizable is more proven for a variety of types of flows. Thus K- realizable has been chosen as the model for flow in the flocculation tank. Below in Figure 4, the stream contours (of the averaged velocity) of the Re=48,000 for the k- realizable model case closely approximate the experimental results.

*Figure 4: Flow over backstep using K-e realizable model*