1. Consider one-dimensional heat conduction. The temperature *T* is governed by the following equation:

$$\frac{d}{dx}\left(k\,\frac{dT}{dx}\right) = h \qquad 0 \le x \le 1$$

Here, k is the coefficient of thermal conductivity and h is heat generation per unit volume. The boundary conditions are:

$$T(0) = 1; \qquad \frac{dT}{dx} = -1\ at\ x = 1$$

For simplicity, take *k* =1 and *h* =1.

   a. Solve this equation analytically.
   b. To solve this equation numerically using the finite-volume (FV) method, derive the discrete equation for interior cells and for cells at the two boundaries.
   c. Divide the domain into 4 cells. Derive the matrix system to be inverted to obtain *T*. Solve this system using MATLAB. Plot the FV solution along with the analytical solution.
   d. Repeat the FV solution with 8 and 16 cells by extending your MATLAB code. Add these two results to the above plot along with a suitable legend. Does the agreement with the analytical solution get better on refining the mesh?

2. Now consider the nonlinear case where the thermal conductivity is a function of temperature i.e. *k* = *k(T)*. The governing equations and boundary conditions are the same as above. Assume the following variation of *k* as a function of temperature:

$$k = 1 + \beta T$$

   Take $\beta = 0.2$. There is no easy analytical solution for this case. The nonlinear term will require that you use an iterative technique in your FV solution. Use *T* = 1 as your initial guess.

   a. Linearize the equation about T_g, a guess value for *T*, by evaluating $k(T)$ as $k(T_g)$. This is called "lagging" the $k(T)$ term and is a simple way to linearize an equation. Indicate how the discrete equation for interior cells and for cells at the two boundaries changes from the linear case.
   b. Divide the domain into 4 cells. Obtain the FV solution using the point-iterative Gauss-Seidel technique. Derive the corresponding update equations to be used for interior and boundary points. Compute the residual using the same procedure as FLUENT (see the *Intro to CFD* handout on Blackboard). Plot the residual vs. iteration. Also, plot the FV solution alongside the analytical solution for the linear case.
   c. Repeat the FV solution with 8 and 16 cells by extending your MATLAB code. Add these two cases to the previous two plots along with a suitable legend. Do you think your FV solution on the finest mesh is approaching the true solution?

Include a printout of your MATLAB codes in your HW solution.