Closed Form Solution for the Semi-Monocoque Shell

Input parameters:

$E = 7.3 \times 10^4$ N/mm^2
$\nu = 0.33$
$L_1 = 750$ mm
$W_1 = 250$ mm
Stiffeners: $H_2 = 15$ mm, $H_3 = 20$ mm, $W_2 = W_3 = 2$ mm
Facesheet: $H_1 = 5$ mm
Pressure $p = 0.05$ N/mm^2
Clamped all around, ¼ symmetry

Deflection

The solution for the clamped-clamped plate is tabulated in Timoshenko and Woinowsky-Krieger, page 202. The maximum deflection is given by

$$w_{\max} = .00256\, p(2W_1)^4 / D$$

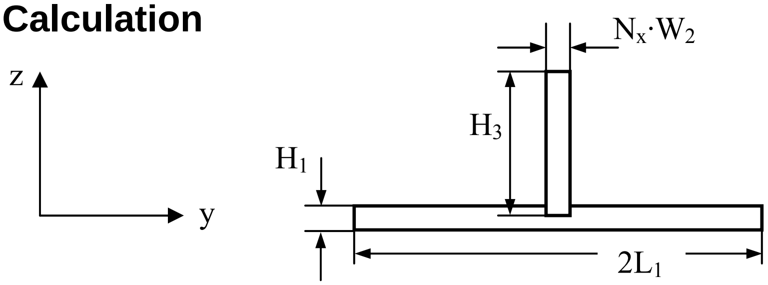
The parameter D is the flexural rigidity of the plate and is defined as

$$D = \frac{E' I}{width}$$

We'll use $E' = \frac{E}{1-\nu^2}$ for facesheets, E=E for stiffeners.

The above equations apply to an isotropic plate. Since the stiffeners in the *x* and *y* directions are different in our case, the effective flexural rigidity *Dx* and *Dy* in the *x* and *y* directions, respectively, are different.

Dx Calculation



(z starts from center of facesheet)

Centroid: We can find the displacement of the centroid due to the stiffeners through a weighted average of the areas.

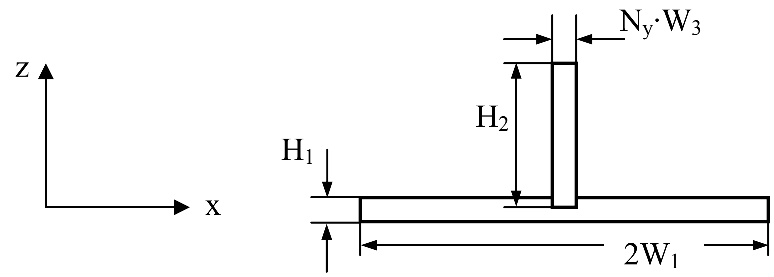
$$\bar{z}_x = \frac{0 \cdot (2L_1 H_1) + (H_3/2) N_x W_2 H_3}{2L_1 H_1 + N_x W_2 H_3}$$

N_x is the number of stiffeners in the *x*-direction. Note that the overlap of the areas is a limitation of shell theory.

***Dx* using parallel axis theorem:**

$$D_x = \frac{E}{1-\nu^2}\left[\frac{H_1^3}{12} + H_1(\bar{z}_x)^2\right] + E\left[\frac{N_x H_3^3 W_2}{12 \cdot 2L_1} + \frac{N_x H_3 W_2}{2L_1}\left(\frac{H_3}{2} - \bar{z}_x\right)^2\right]$$

Dy Calculation



Centroid: $\bar{z}_y = \dfrac{O \cdot (2W_1 H_1) + (H_2/2) N_y W_3 H_2}{2W_1 H_1 + N_y W_3 H_2}$

***Dy* using parallel axis theorem:**

$$D_y = \frac{E}{1-\nu^2}\left[\frac{H_1^3}{12} + H_1(-\bar{z}_y)^2\right] + E\left[\frac{N_y W_3 E_2^3}{12 \cdot 2W_1} + \frac{N_y W_3 H_2}{2W_1}\left(\frac{H_2}{2} - \bar{z}_y\right)^2\right]$$

Plugging in values, we get
Centroids: z_x=0.3101 mm, z_y=0.3435 mm
D_x=2.38e6 N-mm
D_y=2.13e6 N-mm

We'll approximate this as a uniform plate with an average value of D, D=2.253e6 N-mm. Thus, we can use the results for the isotropic plate from Timoshenko and Woinowsky-Krieger.

Stresses: The stresses at the bottom of the facesheet and the top of the stiffener are given by

bot. of facesheet   top of stiffener

$$\sigma_{xx_{\substack{\max\\\min}}} = \frac{E}{1-\nu^2}\frac{M_x}{D_x}\left[\left(-H_1/2-\bar{z}_x\right),\left(H_3-\bar{z}_x\right)\right]$$

$$\sigma_{yy_{\substack{\max\\\min}}} = \frac{E}{1-\nu^2}\frac{M_y}{D_y}\left[\left(-H_1/2-\bar{z}_y\right),\left(H_2-\bar{z}_y\right)\right]$$

Moments:

$M_x = 0.0414\,p(2W_1)^2$ in center

$My = 0.015\,p(2W_1)^2$ in center

$M_x = -0.0831\,p(2W_1)^2$ on clamped edge x = W_1

$My = -0.0571\,p(2W_1)^2$ on clamped edge y = L_1

Results:

Deflection:
w_{max}= 3.55 mm

Stresses:
Pressure is applied to the facesheet from below.

In the center:

bot. of facesheet   top of stiffener

σ_{xx} = [-50, + 350] MPa
σ_{yy} =[-20, +105] MPa

At the clamped edges:

σ_{xx} = [+100, -704]
σ_{yy} = [+78, -403]