Practical Constant-Accuracy Linear Weir

By K. Keshava Murthy,[1] and M. N. Shesha Prakash[2]

Abstract: This paper is concerned with the modifications of the Extended Bell-mouth Weir (EBM weir) earlier designed by Keshava Murthy. It is shown that by providing inclined sides (equivalent to providing an inward-trapezoidal weir) over a sector of a circle of radius R, separated by a distance $2t$, and depth d, the measurable range of EBM can be considerably enhanced (over 375%). Simultaneously, the other parameters of the weir are optimized such that the reference plane of the weir coincides with its crest making it a constant-accuracy linear weir. Discharge through the aforementioned weir is proportional to the depths of flow measured above the crest of the weir for all heads in the range of $0.5R \leq h \leq 7.9R$, within a maximum deviation of $\pm 1\%$ from the theoretical discharge. Experiments with two typical weirs show excellent agreement with the theory by giving a constant-average coefficient of discharge of 0.619.

INTRODUCTION

Linear-proportional weirs that give a linear head-discharge relationship have been extensively studied (Srinivasulu et al. 1970; Venkataraman et al. 1973; Keshava Murthy et al. 1989, 1990, 1991; Ramamurthy et al. 1977). Due to its simple head-discharge relationship, it finds extensive use in irrigation and hydraulic engineering. It will be very useful as a dosing device in chemical engineering and also as a control outlet of grit chambers in sanitary engineering because it nearly maintains a constant-average velocity.

In most proportional weirs, including linear weirs, the datum or reference plane (Reddick et al. 1951) does not coincide with the crest making the accuracy of the weir vary with head, which may be undesirable in a flow-measuring device. In the case of conventional weirs such as rectangular, triangular, and trapezoidal weirs, the datum coincides with the crest, making it a constant-accuracy weir. One such constant-accuracy linear-proportional weir has been designed by Keshava Murthy and Gopalakrishna Pillai (1978).

The exact solutions for the proportional weirs obtained by solving Abel's integral equation result in complex profiles, which, under normal field conditions, pose problems in fabrication. To overcome this, of late, many geometrically simple linear weirs that give near-linear head-discharge relationship within a certain range of heads have been proposed (Venkataraman et al. 1973; Keshava Murthy et al. 1989, 1990, 1991; Ramamurthy et al. 1977). It has been shown that weirs obtained by extending the tangents drawn to the quadrants of the quadrant-plate weirs (called EMB weirs) (Keshava Murthy et al. 1991) possess linear discharge-head characteristics for all heads in the range of $0.2R \leq h \leq 2.15R$ within a maximum deviation of $\pm 1\%$ from the exact-theoretical discharge.

This work extends the linearity range of the EBM without losing its geometrical simplicity by modifying the EBM weir with inclined sides over the sector of a circle (obtained by curtailing the quadrant by $1.5\%R$). In

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addition, the weir is designed to provide constant accuracy. Further, in the case of the EBM weir, the profile is asymptotic to the y axis, which makes it difficult to cut the weir accurately in the neighborhood of the origin leading to erroneous results. This is overcome in the present design by curtailing the depth of the quadrant by $1.5\%R$. The weir parameters, such as linearity range and base-flow depth, are optimized by keeping datum with the crest to achieve maximum range of applicability of the linear characteristics within a specified maximum range of error.

FORMULATION OF PROBLEM

Fig. 1 is a definition sketch of a practical constant-accuracy linear weir. The discharge through a symmetrical sharp-crested weir is defined by the profile $y = f(x)$ as shown in Fig. 1, where x and y are vertical and horizontal axes. Neglecting velocity of approach is given by

$$q = 2C_d\sqrt{2g}\int_0^h \sqrt{h - x}f(x)\,dx \qquad (1)$$

where q = discharge through weir; h = head above crest; g = acceleration due to gravity; and C_d = coefficient of discharge, which is assumed to be constant for streamlined flows through a sharp-crested weir. Similar to conventional weirs, it has to be ascertained through experiments, which may vary within $\pm 1\%$ of the aveage C_d.

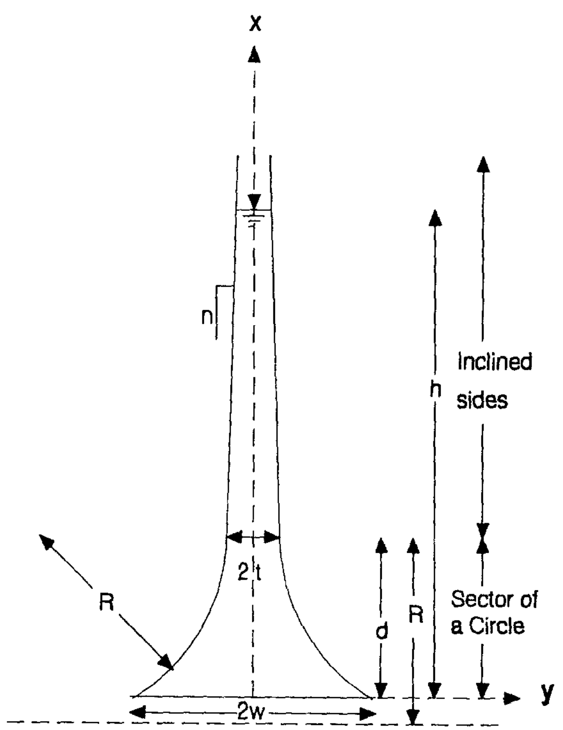

FIG. 1. Definition: Practical Constant-Accuracy Linear Weir

For flows in the base-weir region

$$f(x) = R + t - \sqrt{R^2 - (d - x)^2} \quad (2)$$

and the discharge is given by

$$q = 2C_d\sqrt{2g}\int_0^h \sqrt{h - x}[R + t - \sqrt{R^2 - (d - x)^2}]\,dx; \qquad 0 \le h \le d \quad (3a)$$

When the flow enters the inclined sides (i.e., inward-trapezoidal weir) portion, the discharge is

$$q = 2C_d\sqrt{2g}\left\{\int_0^d \sqrt{h - x}[R + t - \sqrt{R^2 - (d - x)^2}]\,dx + \frac{2}{3}\left[(h - d)^{3/2} + \frac{2}{5n}(h - d)^{5/2}\right]\right\}; \qquad 0 \le h \le h_{\max} \quad (3b)$$

where R = radius of sector of circle; d = depth of sector of circle; t = half-top width of sectors of circle; w = half-crest width of sectors of circle; and n = side slope of inclinded sides.

For convenience, (3a)–(3b) can be expressed in the nondimensional form as

$$Q = \int_0^H [1 + T - \sqrt{1 - (D - X)^2}]\sqrt{H - X}\,dX; \qquad 0 \le H \le D \quad (4a)$$

and

$$Q = \int_0^D \sqrt{H - X}[1 + T - \sqrt{1 - (D - X)^2}]\,dX + \frac{2}{3}\left\{(1 + T)H^{3/2} - \left[1 + \frac{2}{5n}(H - D)\right](H - D)^{3/2}\right\}; \qquad D \le H \le H_{\max} \quad (4b)$$

where $Q = q/KR^{5/2}$; $K = 2C_d\sqrt{2g}$; $H = h/R$; $X = x/R$; $T = t/R$; $W = w/R$; and $D = d/R$. Evaluating these integrals by Simpson's one-third rule, taking a sufficiently small step size, we obtain a high degree of accuracy.

The discharge-head graph, Q versus H, for various values of T and for given values of D and n are shown in Fig. 2. It can be observed that for a wide range of head the plot is nearly linear.

OPTIMIZATION TO OBTAIN LINEAR-DISCHARGE RELATION

The valid range of the replaced head-discharge relation, termed as linearity range, is the difference of upper limit (B) and lower limit (A) as shown in Fig. 3.

$$LR = B - A \quad (5)$$

Consider a point Q_E either above or below the theoretical head-discharge curve adjacent to it. If e is the deviation of Q_E from Q, then

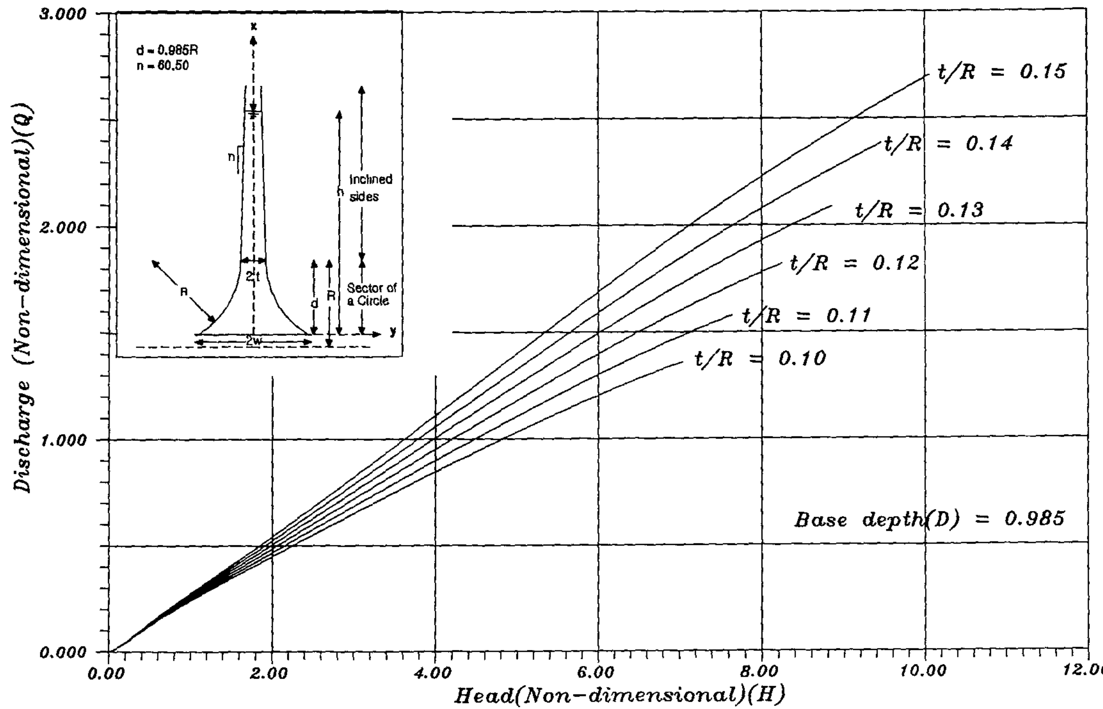

FIG. 2. Theoretical-Discharge Curves

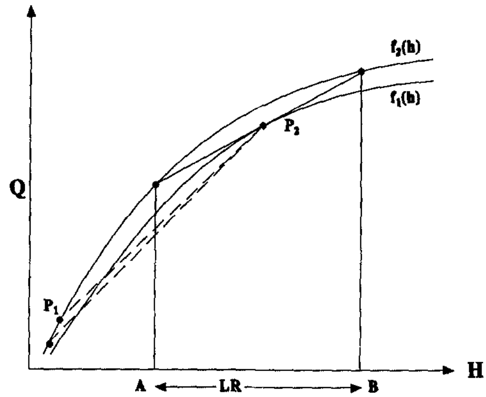

FIG. 3. Optimization Procedure to Obtain Maximum-Linearity Range

$$e = \frac{|Q - Q_E|}{Q} \times 100 \leq E \tag{6}$$

where E = prescribed maximum limit for e [taken as $\pm 1\%$, which is well within the maximum allowable weir-indication error of $\pm 2\%$ (Troskolansky 1960)]. Rearranging (6) we get

$$Q_E = Q(1 \pm e/100) \tag{7}$$

The positive and negative signs in (7) are chosen according to $Q < Q_E$ or $Q > Q_E$. The limits of (7) define the lower and upper-bound curves $f_1(h)$ and $f_2(h)$, respectively, forming a permissible-error region as shown in Fig. 3.

$$f_1(h) = Q(1 - E/100) \tag{8a}$$

and

$$f_2(h) = Q(1 + E/100) \tag{8b}$$

Now the problem is to find the maximum-horizontal projection of the straight-line fit defined by

$$Q_L = mH + C \tag{9}$$

within the bound region defined by the curves (8a)–(8b). A systematic-optimization procedure was developed to obtain the optimum-linearity range as explained in this paper.

A point P_2 is chosen on the extreme right of the $f_1(h)$ curve and joined to a point P_1 on the extreme left of the $f_2(h)$ curve. The entire line P_1P_2 may not be in the region. Point P_1 is moved successively on the $f_2(h)$ curve until the entire line is in the region. The horizontal projection of this line with any possible extension within the region formed by $f_1(h)$ and $f_2(h)$ is then determined. This procedure is repeated for all possible points P_2 on the $f_1(h)$ curve ensuring each time that a straight line inclusive of any extension, which is entirely in the region, is obtained. The line that has the maximum-horizontal projection is selected. The same procedure is repeated by interchanging P_1 and P_2 on $f_1(h)$ and $f_2(h)$ curves to get the global-

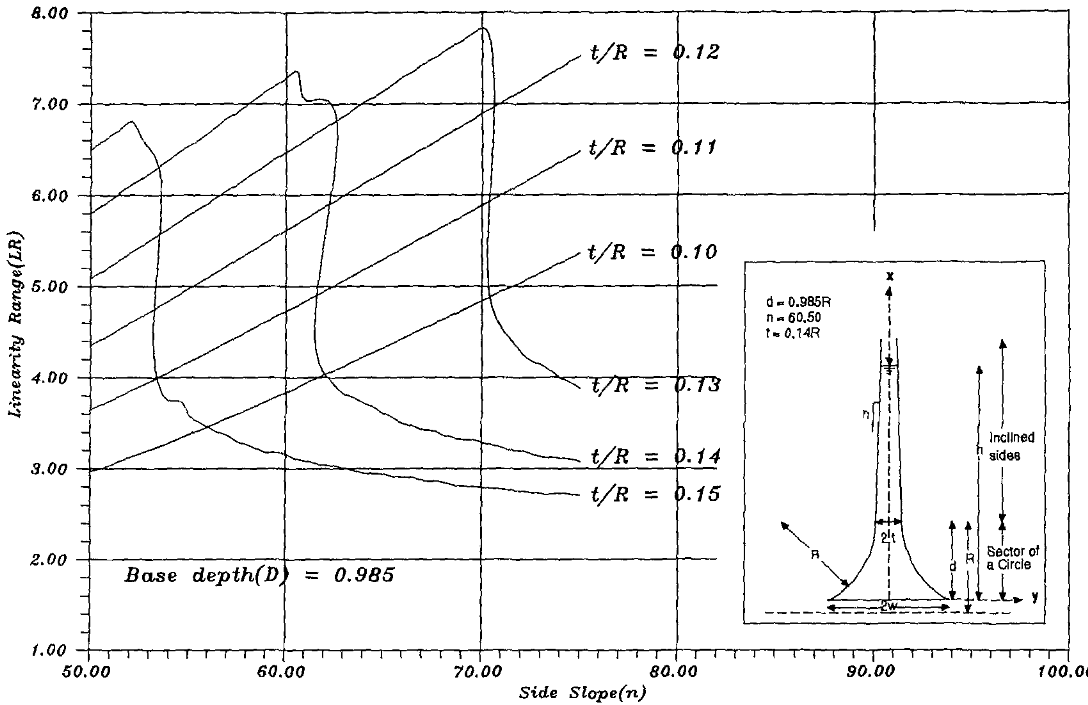

FIG. 4. Variation of LR with n for Given Values of T and D

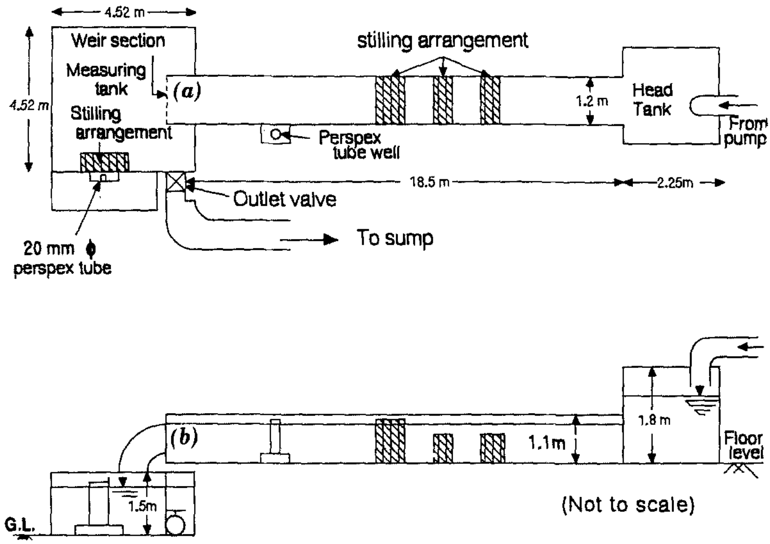

FIG. 5. Laboratory Setup: (*a*) Plan; (*b*) Elevation

maximum-linearity range (*LR*). A computer software in C programming was developed for the aforementioned method on CD4360 Unix system.

ESTIMATION OF WEIR PARAMETERS

Fig. 2 shows a theoretical head-discharge curve for particular values of D and n with various values of T. It is seen that for each value fo T, there exists a distinct head range in which the theoretical head-discharge relation is nearly linear.

To optimize this linear relationship and keep the datum with the crest, the range is calculated for each set of D, n, and T values and plotted. Fig. 4 shows the variation of linearity range with respect to n for particular values of D and T. By comparing the plots, we obtain the optimum parameters of the designed weir as $d = 0.985R$; $t = 0.14R$; and $n = 60.5$. For further values of t/R, the datum lies below the crest as the intercept C is negative.

The proposed linear head-discharge relationship to replace the theoretical one is

$$Q_L = 0.265H; \qquad 0.534 \leq H \leq 7.909 \qquad (10)$$

which can be dimensionally expressed as

$$q_L = 2.3476R^{3/2}h; \qquad 0.534R \leq h \leq 7.909R \qquad (11)$$

EXPERIMENTAL VERIFICATION

Owing to the very large linearity range of the weir, weirs of large heights could not be tested due to the limitations in the experimental setup. Experiments were conducted on two weirs having R = 15 cm and 20.5 cm. The profiles were fabricated very accurately with a nibbling machine using 6.5-mm thick mild-steel plates. The laboratory setup is shown in Fig. 5. The plate weirs were fixed at the end of a rectangular channel 19.5 m long, 1.2

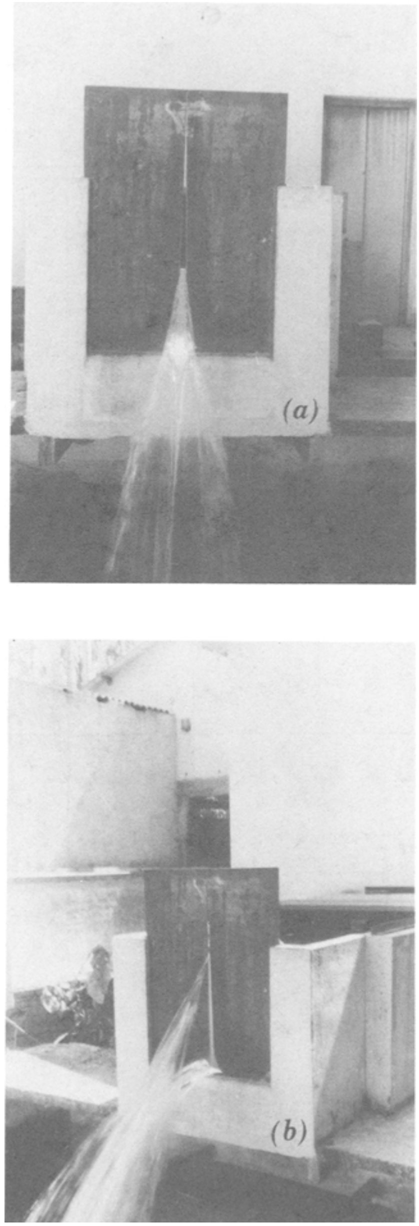

FIG. 6. **(*a*) Front View of Weir Discharging into Tank; (*b*) Side View of Weir Showing Nappe**

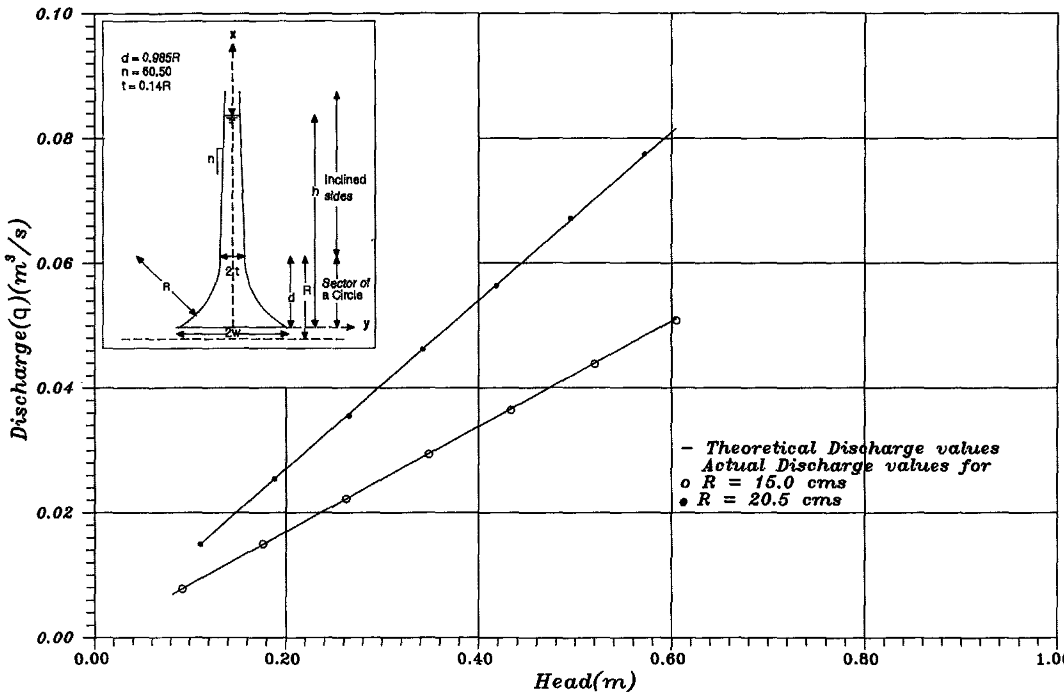

FIG. 7. Experimental Discharge-Head Plot

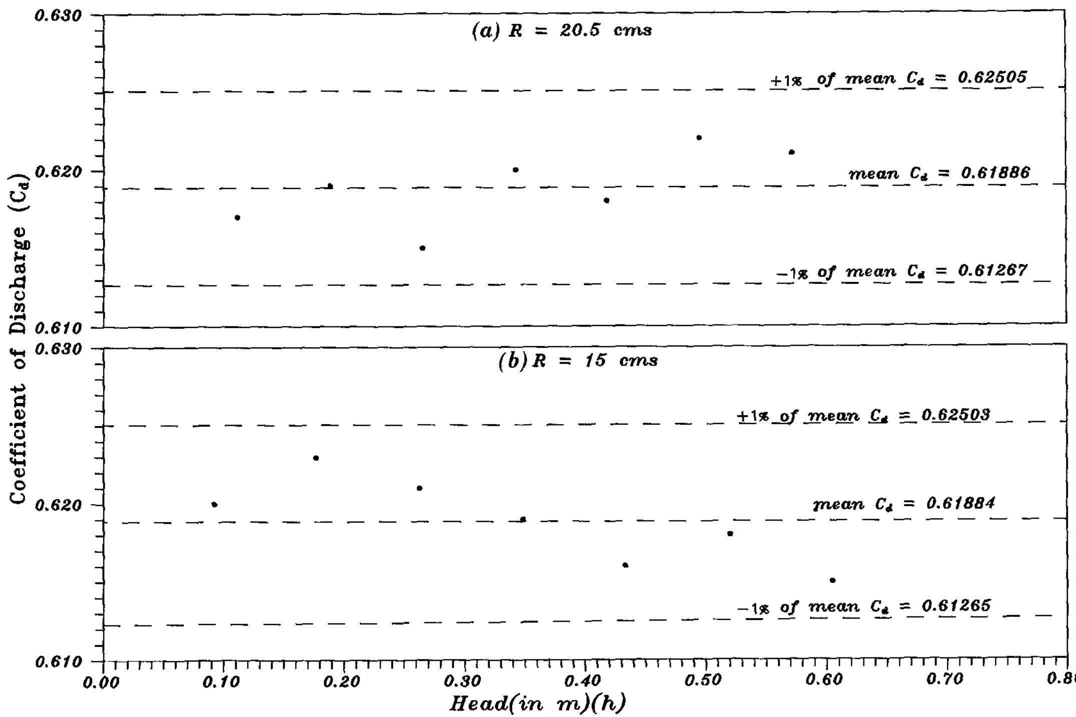

FIG. 8. Variation of C_a with H (in m) of Practical Constant-Linear Weir

m wide, and 1.1 m deep with crest 20 cm above the channel bed. The channel had adequate stilling arrangements. The head over the weir was measured using a point gage with a 0.025 mm resolution, fixed at 4 m upstream of the weir section. Discharges were measured by computing the time taken to collect water in a measuring tank of dimensions 4.52 m × 4.52 m × 1.5 m through readings in a perspex tube of 20 mm internal diameter connected to the bottom of the tank. Fig. 6 shows the front and side views of the discharging practical constant-accuracy linear weir. A plot of the actual discharge versus head measured above the crest is shown in the Fig. 7. The variation of C_d, the coefficient of discharge with head above the crest is shown in Fig. 8. From Fig. 7, it is seen that discharges vary almost linearly with the head in the linearity region fixed by the present analysis. Also from Fig. 8, the C_d corresponding to any head does not vary by more than $\pm 1\%$ of the average C_d, which justifies the assumption of constant C_d in the theoretical analysis.

PRACTICAL APPLICATION

The weir has a simple geometrical shape consisting of circles and straight lines. In addition, the curtailment of the depth of the quadrant at the crest by 1.5% R removes the narrow neck, which would otherwise have formed at the crest, rendering it capable of being fabricated to a very high degree of precision. The ratio $H_{\max}/H_{\min}$ and $Q_{\max}/Q_{\min}$ is also very high (equal to 14.57) making it suitable for large-flow measurements in irrigation. Its simple head-discharge relation and constant accuracy render it a simple-flow-measuring device and a flow recorder. This weir can also be made use of as an outlet weir in grit chambers to maintain a near-constant-settling velocity (Keshava Murthy et al. 1968).

CONCLUSIONS

It is shown that the weir with sectors of a circle of radius R and depth d followed by inclined sides separated by a distance $2t$ and inward sideslopes n, can be used to give a linear head-discharge relationship within a maximum deviation of $\pm 1\%$ from the theoretical discharge, over a range of heads determined by the values of t/R, d/R, and n.

The weir parameters (namely, linearity range, base flow depth, and datum constant) vary with the dimensions of the weir such as d, t, and n.

With the optimum values of $t/R = 0.14$; $d/R = 0.985$; and $n = 60.5$, the discharge through the weir is proportional to the depths measured above the crest of the weir for all heads in the region $0.543R \le h \le 7.909R$.

The linearity range of the Extended Bellmouth Weir is enhanced by more than 375%.

The datum of the weir lies with the crest, thereby making it a constant-accuracy linear weir.

The curtailment of the quadrant depth by 1.5%R renders it easy to fabricate the exact profile and thereby avoids error in experiments.

Experiments are in very good agreement with the theory by yielding a constant-average coefficient of discharge of 0.619.

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APPENDIX I. DESIGN EXAMPLE

Let us assume that the aforementioned weir is to be designed for a channel of width 75 cm with a discharge of 0.25 m^3/s and the C_d considered as 0.619.

Solution

The data used in the design are $q = 0.25$ m^3/s and $C_d = 0.619$. From (11)

$$q_{\text{Lact}} = C_d \times 2.3476R^{3/2}h; \qquad \ldots 0.534R \le h \le 7.909R \tag{12}$$

The maximum-measurable head is $7.909R$, which has to correspond to the maximum discharge 0.25 m^3/s. Substituting for qL and C_d in (12) we get

$$0.25 = 0.619 \times 2.3476R^{3/2} \times 7.909R \tag{13}$$

hence $R = 21.63$ cm. Therefore, the dimensions and other parameters of the practical constant-accuracy linear weir are radius of sector of circle $R = 22$ cm; depth of sector of circle $d = 0.985R = 21.67$ cm; half-top width of sectors of circle $t = 0.14R = 3.08$ cm; base width $= 2w = 2(t + 0.8775R) = 44.78$ cm; side slope of the inward trapezium $n = 60.5$; maximum-measurable head $h_{\max} = 7.909R = 174$ cm; base-flow depth $= 0.534R = 11.77$ cm; linearity range $= 7.375R = 162.25$ cm; and minimum-measurable range $= q_{\max} = 0.25/14.57 = 0.01716$ m^3/s.

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APPENDIX III. NOTATION

The following symbols are used in this paper:

A = base-flow depth or lower limit of linearity range;
B = upper limit of linearity range;
C_d = coefficient of discharge;
c = intercept constant;
E = prefixed maximum percentage of error;
e = deviation of Q_E from Q;
$f_1(h), f_2(h)$ = curves defining permissible region for Q_E to lie in Q versus H plot;
g = acceleration due to gravity;
H = h/R;
$H_{\max}, H_{\min}$ = nondimensional heads at upper and lower limit of linearity range, respectively;
h = head above weir crest;
K = $2C_d\sqrt{2g}$, a dimensional constant;
LR = linearity range;
m = slope constant;
n = side slope of inclined sides of weir;
Q = $q/KR^{5/2}$;
Q_L = nondimensional replaced linear-discharge relationship;
$Q_{\max}, Q_{\min}$ = nondimensional discharges at upper and lower limit of linearity range, respectively;
q = discharge;
q_L = proposed linear relationship;
q_{Lact} = actual discharge computed from q_L;
R = radius of sector of circle;
T = t/R;
t = half-top width of base weir (sector of circle);
W = w/R;
w = half-crest width of practical constant-accuracy-linear weir;
X = x/R;
x = vertical coordinate;
Y = y/R; and
y = horizontal coordinate.