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3	CHARACTERIZATION OF FLOCS AND FLOC SIZE
4	DISTRIBUTIONS USING IMAGE ANALYSIS
5	
6	A Thesis
7	Presented to the Faculty of the Graduate School
8	of Cornell University
9	in Partial Fulfillment of the Requirements for the Degree of
10	Master of Science
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14	by
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ABSTRACT

33 A nonintrusive digital imaging process was developed to study particle size distributions created 34 through flocculation and sedimentation. This process was calibrated by measuring standardized 35 polystyrene particles of known size and was utilized to count and measure individual kaolin clay 36 particles as well as aggregates formed by coagulation with polyaluminum chloride and 37 flocculation. Identification of out-of focus flocs was automated with LabVIEW and used to 38 remove them from the database that was analyzed. The particle diameter of the test suspension of 39 kaolinite clay was measured to be 7.7±3.8 µm and a linear relationship was obtained between 40 turbidity and the concentration of clay particles determined by imaging. The analysis technique 41 was applied to characterize flocs and floc particle size distribution as a function of coagulant 42 dose. Removal of flocs by sedimentation was characterized by imaging and the negative 43 logarithm of the fraction of turbidity remaining after settling had a positive linear association 44 with the logarithm of aluminum dose. The maximum floc size observed in the settled water was 45 less than $120 \,\mu\text{m}$, which was in accordance with the value predicted by a terminal velocity model 46 for the capture velocity of the experimental tube settler of 0.21 mm/s.

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BIOGRAPHICAL SKETCH

51	Siwei Sun was born in Shanghai, China in 1990. She graduated with a degree in Environmental
52	Science and Engineering from Shanghai Jiao Tong University in 2013. In college, she was a
53	member of a student group BLUE SKY and worked to enhance public environment
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57	towards the M.S. Degree in Environmental Engineering.
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187 **LIST OF ABBREVIATIONS**

188	Ι	the matrix of the original image pixel values
189	I _f	the sobel filter of the image matrix
190	NTU	Nephelometric Turbidity Unit
191	PACl	Polyaluminum chloride
192	ROI	Region of interest
193	SSE	sum of squared errors of prediction
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195		
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201		

204	A _{flowcell}	cross sectional area of the flow cell (L^2)
205	A _{pixel}	projected area of particles in pixels
206	A_{tube}	cross sectional area of flocculator tube (L^2)
207	C_{Al}	Al concentration of coagulant stock (M/L ³)
208	C _{clay}	clay concentration added to raw water (M/L ³)
209	$C_{effluent}$	fraction of the effluent turbidity caused by the floc size class
210	C _{influent}	fraction of the influent turbidity caused by the floc size class
211	C _{plant}	Al dose within the flocculator (M/L^3)
212	D	inner diameter of the flocculator tube (L)
213	D _{fractal}	3-D fractal dimensión
214	D_p	2D fractal dimension
215	D_{v}	3D fractal dimension described by Maggi and Winterwerp
216	d	floc diameter (L)
217	d _{clay}	diameter of primary particle (L)

218	d_{pixel}	spherical-equivalent diameter in pixels		
219	G	velocity gradient (1/t)		
220	$G_{CampStein}$	velocity gradient described by Camp and Stein (1/t)		
221	Ē	average velocity gradient (1/t)		
222	$\overline{G_c}$	average velocity gradient in figure eight flocculator (1/t)		
223	$\overline{G_{Gregory}}$	velocity gradient described by Gregory (1/t)		
224	H _{image}	depth of field (L)		
225	k	total floc number		
226	L _{image}	height of the image (L)		
227	L _{tube}	length of the flocculator tube (L)		
228	$L_{tube\ settler}$	length of tube settler (L)		
229	l	length scale of the sobel filter kernel (L)		
230	l _{pixel}	pixel size (L)		
231	NA	numerical aperture		
232	n_0	number of primary particles counted in a 2D image		
233	n_i	number of primary particle in floc <i>i</i>		
234	n _{total}	total number of primary particles within sample volume		

235	Р	power input $(M \cdot L^2/t^3)$
236	P _{pixel}	perimeter of flocs (in pixels)
237	<i>pC</i> *	removal efficiency
238	Q_{Al}	flow rate of coagulant solution (L^3/t)
239	$Q_{flowcell}$	flow rate in flow cell (L^3/t)
240	Q_{plant}	flow rate of the plant (L^3/t)
241	R	radius of the pipe (L)
242	R _c	diameter of curvature of the flocculator coils (L)
243	r	radial distance from the pipe axis (L)
244	S	inner width of the tube settler (L)
245	t	operation time (t)
246	V	volume of the pipe (L^3)
247	V _{clay}	clay volume measured according to the images (L^3)
248	v_0	the maximum velocity in the fluid
249	$v_{capture}$	capture velocity of tube settler (L/t)
250	<i>v</i> _r	velocity of fluid at a radial distance r from the pipe axis (L/t)
251	v_t	floc terminal velocity (L/t)

252	$v_{tube\ settler}$	velocity inside tube settler (L/t)
253	v_{up}	vertical component of the velocity in the tube settler (L/t)
254	$ar{ u}$	average velocity of fluid (L/t)
255	W _{image}	width of the image (L)
256	x	dimensionless floc size
257	α	normalized minimum pixel value
258	α_t	threshold value of normalized pixel value
259	β	dimensionless maximum floc image intensity gradient
260	β_t	threshold value of image intensity gradient
261	Δp	pressure drop along the pipe $[M/(L \cdot t^2)]$
262	ε	average energy dissipation rate (L^2/t^3)
263	θ	angle of the tube settler
264	λ	wavelength of incident light (L)
265	μ	dynamic viscosity of fluid $[M/(L \cdot t)]$
266	ν	kinetic viscosity (L^2/t)
267	v_{H_2O}	kinematic viscosity of water (L^2/t)
268	$ ho_{clay}$	density of primary clay particle (M/L ³)

 ρ_{H_2O} density of water (M/L³)

 Φ shape factor for drags on flocs

271 CHAPTER 1: INTRODUCTION

272 The quality of water has a significant impact on both human health and socioeconomic 273 development. The criteria for access to sufficient water for domestic uses include, but are not 274 limited to the following: safety, reliability, sustainability, affordability and physical accessibility. 275 However, it is estimated by the United Nations (Millennium Development Goals Report, 2012) 276 that 783 million people, or 11% of the global population, do not have access to improved sources 277 of drinking water (such as house hold connections and public standpipes). In some rural areas, 278 even higher portions of the population lack access to improved drinking water and are exposed to 279 dangerous levels of biologically or chemically contaminated water due to inadequate water 280 treatment systems. Thus, technology to provide safe water for hundreds of millions of people at 281 low cost is in demand.

282 AguaClara is a multi-disciplinary program in the School of Civil and Environmental Engineering 283 at Cornell University that conducts laboratory research leading to the design of sustainable, 284 gravity-powered, electricity-free water treatment plants. The treatment processes include rapid 285 mix, flocculation, sedimentation, filtration and disinfection. AguaClara designs of water 286 treatment plants constructed in Honduras provide clean water that meets the guidelines of the 287 World Health Organization in a cost-effective way. Ten communities consisting of 40,000 288 people are served by AguaClara technologies and it is anticipated that more people will benefit 289 from this program in the near future (AguaClara, 2015).

290 The research in this thesis is a part of AguaClara program and it presents development of a non-291 intrusive imaging process as a tool for measurement of flocs formed through flocculation and 292 clarified by sedimentation. Natural water sources contain inorganic and organic particles 293 including pathogens, which are harmful to human health. Flocculation and sedimentation are 294 important parts of drinking water treatment in that they can remove these particles. Flocculation 295 is a process where colloids aggregate and form flocs that are removed by gravity forces in the 296 sedimentation tank. However, many of the fundamental mechanisms that control flocculation are 297 not well understood. For example, experimental data from the AguaClara research team suggests 298 use of a mechanistic model where small particles do not readily attach to big flocs. There appears 299 to be something about the collisions between particles that are very different in size that makes 300 aggregation difficult. Observation of floc collisions may inform our understanding of factors 301 that inhibit aggregation.

302 The objectives of this research were to develop non-destructive imaging techniques that permit 303 floc characterization and to study the influence of coagulant dose on floc size distribution and 304 turbidity removal. The variances in particle size distributions under different operating 305 conditions can be of use in the understanding of aggregation mechanisms. Understanding how 306 floc size distribution influences the flocculation process and removal of particles by 307 sedimentation will contribute to the optimization of treatment plant design. The development of 308 the image analysis tool also lays a foundation for future studies of particle collisions and the 309 mechanisms that control aggregation.

310

CHAPTER 2: CHARACTERIZATION OF FLOCS AND FLOC SIZE DISTRIBUTION¹

314 **2.1 Abstract**

315 A nonintrusive digital imaging process was developed to study particle size distributions created 316 through flocculation and sedimentation. This process was calibrated by measuring standardized 317 polystyrene particles of known size and was utilized to count and measure individual kaolin clay 318 particles as well as aggregates formed by coagulation with polyaluminum chloride and 319 flocculation. Identification of out-of focus flocs was automated with LabVIEW and used to 320 remove them from the database that was analyzed. The particle diameter of the test suspension of 321 kaolinite clay was measured to be $7.7\pm3.8 \,\mu\text{m}$ and a linear relationship was obtained between 322 turbidity and the concentration of clay particles determined by imaging. The analysis technique 323 was applied to characterize flocs and floc particle size distribution as a function of coagulant 324 dose. Removal of flocs by sedimentation was characterized by imaging and the negative 325 logarithm of the fraction of turbidity remaining after settling had a positive linear association 326 with the logarithm of aluminum dose. The maximum floc size observed in the settled water was

¹The content of this chapter will be submitted to *Environmental Engineering Science*, with coauthors M. L. Weber-Shirk and L. W. Lion.

327 less than 120 μ m, which was in accordance with the value predicted by a terminal velocity model 328 for the capture velocity of the experimental tube settler of 0.21 mm/s.

329 2.2 Introduction

Flocculation facilitates aggregation of inorganic and organic particles in water sources, and is a
 crucial pretreatment process prior to particle removal by sedimentation and filtration. The fluid
 velocity gradient (G) is widely recognized as a key design parameter for laminar flow

333 flocculators. Many studies have shown a relation between floc sizes and G (Park et al., 1972;

Matsuo and Unno, 1981; Hopkins and Ducoste, 2003). Gregory (1981) and Camp and Stein

335 (1943) proposed two different ways to calculate G, in terms of a given flow rate and tube

diameter. For the conditions of this research, the average velocity gradient calculated by Camp

and Stein (70.6/s) is 6% higher than that by Gregory (see calculations in Appendix A.). Energy

dissipation rate (ϵ) is also used in the design of water treatment plants and, under conditions of laminar flow, is related to \overline{G} , as follows (Coufort et al., 2008):

$$340 \quad \overline{G} = \sqrt{\frac{\varepsilon}{\nu}} \tag{1}$$

341 Where, ε is the average energy dissipation rate, and

342 v is the kinematic viscosity of fluid.

During flocculation, particle sizes, structures and shapes can all affect aggregation behavior and
 collision efficiency (Jiang and Logan, 1991). Yao et al. (2014) reported that concentration of
 particles less than 5 µm is in a positive linear relationship with water turbidity in the range of 0-

40 Nephelometric Turbidity Units (NTU). Nan et al. (2009) suggested that flocs in different size
ranges contribute differently to the decrease in turbidity after sedimentation. Thus, the
measurement of particle size distribution during flocculation can be of use in understanding
particle removal subsequent to flocculation.

350 Particle size characterization can be accomplished using a Coulter counter (Zhang et al., 2007) or 351 by the electrical sensing zone method (Gibbs, 1982). However, both analyses require withdrawal 352 of samples from a suspension that may disrupt fragile flocs (Chakraborti et al., 2000). Some 353 researchers have adopted photographic techniques and image analysis as a non-invasive tool for 354 the continuous measurement of changes in floc sizes in jar tests. Bouyer et al. (2004) used a laser 355 beam as light source and VISILOG 5 for image analysis to obtain the instantaneous size 356 distribution of flocs. However, they found it difficult to exploit the data without analyzing the 357 contour of particles because there were too many possible intersections between the laser plane 358 and particle shapes, and particle data needed to be discarded if the contour of the particle was 359 shaded. PIV (particle image velocimetry) software for image acquisition and storage and NIH-360 Image software for image analysis have been used by Chakraborti et al. (2000) to characterize 361 alum flocs. Keyvani and Strom (2013) developed a fully automated image processing script to 362 remove out-of focus particles to attain more precise size distributions with Image J and 363 MATLAB.

Based on Keyvani and Strom's study, an image analysis script was developed in this research
using National Instruments LabVIEW and Vision Builder Toolkit to explore particle size
distribution changes during flocculation and sedimentation. The LabVIEW software
incorporates image acquisition and analysis.

368 2.3 Experimental Methods

369 2.3.1 Flocculator Setup

Figure 1 shows a schematic of the laboratory apparatus. Aerated water was pumped from a 370 371 temperature-controlled reservoir and mixed with a concentrated kaolinite (R.T. Vanderbilt Co., 372 Inc. Norwalk, CT.) clay stock to form synthetic raw water. Raw water turbidity was controlled 373 by adjusting the flow rate of clay stock (see equation 25 in the Appendix B.) and was 374 continuously measured using a MicroTOL 3 turbidity meter (Model number: 20055, HF 375 Scientific, Inc. FT. Myers, FL.). The turbidity meter was equipped with a flow cell so that there 376 was no need to withdraw samples from the raw water. Polyaluminum chloride (PACl) coagulant 377 doses (Holland Company. Adams, MA.) ranging from (0.53 to 2.65 mg/L as Al) were mixed into 378 the raw water. Flocculation was accomplished by laminar flow through a coiled 9.52 mm (inner 379 diameter) tube. The average energy dissipation rate of the flocculator was 5 mW/kg and the 380 hydraulic residence time was 300 s. Sedimentation occurred in a tube settler with a capture 381 velocity (also referred to as a critical velocity) of 0.21 mm/s. Approximately 8% of the 382 experimental flow could be directed to a flow cell. Two valves were utilized to control the type 383 of water entering the flow cell allowing imaging of either flocculated water or settled water. 384 Effluent turbidity was continuously measured using MicroTOL 2 turbidity meter (Model number: 385 20053, HF Scientific, Inc. FT. Myers, FL.). Process Controller software created using LabVIEW 386 by Weber-Shirk (2008) was utilized for acquisition of turbidity data.







Figure 1. Schematic of the experimental apparatus.

389 In laminar flow, there is no turbulence to resuspend particles that may settle in the flocculator.

- 390 As a result, the experimental flocculator tubing was coiled in a figure eight configuration to
- 391 create a secondary flow circulation to prevent floc sedimentation (Tse et al., 2011).
- 392 The average velocity gradient ($\overline{G_c}$) in the coiled figure eight flocculator that accounts for the
- 393 secondary flow was calculated as described by Tse et al. (2011):

$$394 \quad \overline{G_c} = \overline{G} \sqrt{1 + 0.033 \left[log \left(\frac{4Q_{plant}}{\pi D \nu} \sqrt{\frac{D}{R_c}} \right) \right]^4}$$
(2)

395 Where Q_{plant} is the experimental flow rate,

- 396 *D* is the inner diameter of the flocculator tube, and
- R_c is the diameter of curvature of the flocculator coils.

398 2.3.2 Imaging system

The camera system (see Figure 2) consisted of an LED light source and a Flea3 FL3-GE-13S2M monochrome GigE camera (Point Grey Research, Inc. Richmond, BC, Canada) controlled by the LabVIEW program. The camera was a 1288×964 pixel progressive scan, monochrome 1/3" CCD fitted with an M Plan Apo 10× infinity-corrected objective lens with a numerical aperture of 0.28 (Mitutoyo Corporation, Japan). The camera can capture continuous images at up to 31 frames per second or single images by external trigger or via software control.



405

 Figure 2. Imaging system consisting of LED light, CCD camera attached to a computer and the suspended sample in a flow cell.

408 Based on the camera sensor format and 10× magnification of the objective lens, the field of view 409 for the imaging system was 480 µm×360 µm. Each pixel sampled an area in the field of view of 410 0.375 µm by 0.375 µm. The depth of field of the objective lens was calculated over a range of 411 influent turbidities and the average value was $500 \pm 90 \,\mu\text{m}$ (See equation (11) in part 2.6.1). The 412 constraints for maximum floc size measurements are the field of view and the depth of field of 413 the lens. For flocs smaller than this depth of field, it is likely that the entire floc will be in focus. 414 Since the size of one pixel is close to the wavelength of visible light (approximately 400-700 nm) 415 (Pal and Pal, 2001), the diffraction of light can result in airy disks around particles in images and 416 errors in particle size measurements. An airy disk is a bright central core surrounded by 417 diffraction rings. More than 80% of light energy concentrates in the central ring of the airy disk 418 (Greivenkamp, 2004), as the intensity distribution in Figure 3(A) shows.



Figure 3. Airy disks. (A) Airy pattern and intensity distribution. (B) Airy patterns around a particle.

419

422 A consequence of the formation of airy disks is that a point in an object will not be imaged as a

423 spot with sharply defined edges. Instead, it is imaged by the objective lens as a spot surrounded

by diffraction rings, which can affect the accuracy in measuring particle diameters. Figure 3(B)
illustrates one example showing the airy patterns around a particle. It is obvious that the edges of
the floc are not sharply contrasted.

427 The radius of the central ring of the airy disk can be calculated by ("Wavelength effects on428 performance", 2015),

$$429 \quad r_{airy\,disk} = \frac{0.61\lambda}{NA} \tag{3}$$

430 Where λ is the wavelength of the incident light,

431 *NA* is numerical aperture, and was approximately 0.28 for the objective lens used in this432 research.

The estimated radius of airy disk caused by yellow light ($\lambda \approx 590nm$) is around 1.29 µm. Since each point around the particle has airy pattern, particles less than $1.29 \times 2 \approx 2.6$ µm in diameter (particle area of approximately 39 pixels) cannot be clearly identified. The apparatus could thus measure particle sizes ranging from 2.6 µm (set by the airy disk) to more than 300 µm (set by the field of view). Standardized particles of known size were utilized to determine the error in particle size measurement caused by light diffraction.

439 The camera was connected to the computer via Gigabit Ethernet, which allowed an acceptable

440 transfer speed of 100 MB/s (equivalent to 2226 images of JPEG format per second). The camera

441 was mounted on a horizontal translation stage fixed to an aluminum platform. An LED light

442 provided bright field illumination of flocs in the flow cell (Keyvani and Strom, 2013).

The flow cell was constructed from a glass cuvette with a cross sectional area of $1 \text{ cm} \times 1 \text{ cm}$. The inlet and outlet of the flow cell had a diameter of 7.1 mm. The flow rate inside the flow cell was constrained by the minimum shutter speed of the digital camera. It was assumed that blurry images could occur when a pixel moved 1/4 of its length. Therefore, the maximum flow rate inside the flow cell was calculated by equation (4).

448
$$Q_{flowcell} = 0.375 \,\mu m \times \frac{1}{4} \times \frac{A_{flowcell}}{t} \tag{4}$$

449 Where, $A_{flowcell}$ is the cross sectional area of the flow cell and t is the time the shutter is open 450 (33 µs).

The flow rate through the sample cell was set to 0.28 mL/s based on equation (4). The average velocity gradient in the inlet port of the flow cell was 5.4/s making the average velocity gradient entering the flow cell less than 7.6% of the average velocity gradient in the flocculator. The average velocity of the water flowing through the flow cell was 2.84 mm/s. This velocity was also much higher than the sedimentation velocity of the largest flocs measured.

456 During initial testing of the imaging system the optimal shutter speed for image contrast was 457 determined to be 330 μ s or 10 times longer than the minimum shutter speed. The particle travel 458 distance during this time is 1 μ m and there was no evidence of significant image blurring. The 1 459 μ m travel distance during the time when the shutter is open is small compared to the minimum 460 particle size of 2.6 μ m.

461 **2.4 Image analysis**

462 The image analysis script accomplished four functions: (1) reduction of image noise, (2)
463 identification of particles from background, (3) removal of particles that were out of focus or that
464 had portions beyond the image border, (4) calculation and recording of particle sizes. The image
465 processing functions prepackaged in LabVIEW are capable of identifying and measuring
466 particles. These functions include filters, threshold, basic or advanced morphology and particle
467 analysis ("Image analysis and processing," 2008).

468 2.4.1 Identification of particles

A filter was first applied to each image to reduce small changes in pixel values caused by variability in the charge-coupled device of the camera. The Gaussian filtering function of the LabVIEW vision application was found to work best at reducing noise by attenuating the variations of grey scale intensity in a pixel's neighborhood. The Gaussian filter effectively smoothed the fuzzy edge of the particles in the image so that one could better extract useful information from a particular image.

Filtering was followed by the operation of thresholding, which distinguished particles from the background and produced a binary image with 0 representing the background and 1 representing particles. In general, there are two thresholding methods: global thresholding and local thresholding. Global thresholding identifies particles based on a single grayscale value. In local thresholding, each pixel is categorized based on the intensity of pixels in its neighborhood ("Thresholding," 2013). Global thresholding usually requires a specified threshold range for each 481 set of tests, while local thresholding can identify particles automatically. Thus, background 482 correction ("Thresholding", 2013) within local thresholding function was utilized in the image 483 analysis procedure in this research in that this technique is well suited for conditions where 484 images exhibit nonuniform light intensities caused by other out of focus particles in the 485 background.

486 Figure 4 shows the application of local thresholding (background correction). This local487 thresholding method appears to function well in particle recognition.





488

Figure 4. Application of local thresdholding. (A) The original image with flocs. (B) Local thresholding applied to image A.

When local thresholding is applied to an image, holes and gaps inside a floc can arise due to the
complicated structure of the aggregates. The holes and gaps must be filled to calculate the
particle area. Thus, some morphological transformations were utilized to prepare particle images
for quantitative analysis. These transformations included closing the object perimeter, filling
holes, and removing particles touching the border as well as small particles less than 39 pixels
(equivalent spherical diameter of 2.6 µm). Particles less than 39 pixels were not considered

497 because they were too small to obtain an accurate measurement of their shape and area as a result498 of airy disc patterns.

The closing objects function was able to fill small holes and smooth the boundaries of the floc.
These changes only slightly alter the shape or the area of the object. The filling holes function
filled any remaining holes inside the particle boundary. Figure 5 shows an example of the
morphology transformation functions.



503

Figure 5. Example of morphological transformation. (A) Original grey scale image, (B)
 image after background correction and closing objects, (C) image after filling holes, (D)
 image after removing small particles or particles that touched the border of the image.

507 The next step in image analysis was to measure the area and the coordinates of the bounding

508 rectangle of each floc in pixels. The spherical–equivalent diameter in pixels can be calculated as,

509
$$d_{pixel} = \sqrt{\frac{4A_{pixel}}{\pi}}$$
(5)

510 Where A_{pixel} is the projected area of the particle, in pixels. As mentioned above, particle sizes

511 were also calibrated in this step to account for the influence of airy disks.

512 The Region of Interest (ROI) for each floc was defined by the coordinates of the bounding

513 rectangle. The floc ROI of the original image was used to assess if the floc was in focus.

514 **2.4.2 Removing out-of focus particles**

515 As noted above, local thresholding could identify almost all particles within an image regardless

516 of their degrees of focus, except for some extremely blurry flocs, such as the ones in Figure 4(A).

517 Hence, the next part of the image analysis script acted to remove out-of focus flocs.

518 Whether an object in an image appears blurry or not is determined by its focus quality

519 characterized by the sharp differences between background and object edges (Klinger, 2003).

520 Keyvani and Strom (2013) introduced a concept of "clarity value" index to determine the focus

521 quality of each floc and thus distinguish in-focus particles from the blurry ones. In their work,

522 each image was treated with a convolution of a first Gaussian kernel in both horizontal and

523 vertical directions. The maximum value of the filtered image associated with each floc could be

524 used to define how close the floc was to the focal plane.

Flocs that are in focus have sharp gradients between the background and the floc. Flocs that are out of focus have weaker gradients at their boundaries. The image gradient intensity at the floc boundaries was used to eliminate flocs that were not in focus. The sobel filter computes an approximation of the image intensity gradient. High sobel filter values indicated the floc was in focus. The maximum image intensity gradient was computed for each floc. In order to eliminate the effect of LED light intensity and shutter speed (which will have influence on the light

531 intensity), the maximum image intensity gradient was divided by the mean pixel value of the

532 whole image. The result was then multiplied by a length scale $(3 \times 0.375 \mu m)$ related to kernel size

533 (a kernel is a 3×3 matrix for a sobel filter) to give in a dimensionless parameter used to

534 discriminate between in focus and out of focus flocs.

535
$$\alpha = \frac{\min[I(ROI_i)]}{mean[I(ROI_i)]}$$
(6)

536
$$\beta = \frac{\max[I_f(ROI_i)] \times l}{mean[I(ROI_i)]}$$
(7)

- 537 Where, α is the normalized minimum pixel value,
- 538 *I* is the matrix of the original image pixel values,
- 539 ROI_i is the bounding rectangle of floc *i*,
- 540 β is the dimensionless maximum floc image intensity gradient,

541
$$I_f$$
 is the sobel filter of the image matrix,

542 l is the length scale of the sobel filter kernel, which is 1.1 µm.

543 Darker flocs (which have smaller pixel values) were closer to the focal plane. Thus, the

544 minimum pixel value (α) of I (the original image matrix) associated with each floc was also

545 measured to assist in the determination of focus quality. Some transparent particles of unknown

- 546 origin were observed in the clay mixture with a β greater than 0.16. These unknown particles
- 547 were discarded by setting a minimum pixel value of I. The minimum pixel value of the original
- 548 image was normalized to be dimensionless as described above.

549 Threshold values for both α and β were used to distinguish in-focus particles from those which

- 550 were not in focus and were determined by examining computed values from a great number of
- images. After comparison, the image intensity gradient threshold value (β_t) was set to be 0.16
- and the threshold value of the normalized minimum pixel value (α_t) to be 0.56. Therefore,
- particles with β above 0.16 and α below 0.56 were considered as in focus flocs and the remaining
- flocs were removed from the database. The calculated spherical-equivalent diameters of those in-
- 555 focus flocs were then written to a cvs file for each image.
- 556 Figure 6 and Figure 7 are two examples showing different α and β values and the focus quality
- 557 within an image.



558

559

Figure 6. Sample image of identified and measured flocs.

- 560 For Figure 6, three flocs are identified after thresholding (actually four particles were identified,
- 561 however one touching the top border was removed). Floc 1 has the best focus quality; flocs 2 and

562 3 may possibly be in-focus. As is shown in Table 1, a value of $\beta_t = 0.16$ and a value of $\alpha_t = 0.56$

563 worked well as a particle filter, removing the out-of focus flocs and retaining the in-focus ones.

Table 1. Identified flocs in Figure 6 with their associated α and β values. Bold values meet
 the constraints.

Floc number	α	α β Accepta		ble Spherical diameter (µm)	
1	0.26	0.69	Yes	56.4	
2	0.65	0.15	No		
3	0.37	0.20	Yes	6.9	

566 Figure 7 is another example showing how the algorithm performed. There are seven particles

567 detected in the image.



568



570 However, by visual observation, one could easily conclude that there is only one in-focus floc

571 (floc 4), which agrees with the result in Table 2.

0.38

0.74

0.66

0.76

573	the constraints.						
	Floc number	α	β	Acceptable	Spherical diameter (µm)		
	1	0.69	0.13	No			
	2	0.69	0.19	No			
	3	0.48	0.09	No			

0.41

0.27

0.08

0.04

Yes

No

No

No

11.8

Table 2. Identified flocs in Figure 7 with their associated α and β values. Bold values meet the constraints.

574 Figure 8 summarizes the order of operations performed on each image to obtain the geometric

575 characteristics of particles.

4

5

6




Figure 8. Flowchart of image analysis procedure.

579 2.5 Terminal velocity

580 Terminal velocity is the velocity of a floc when the forces of gravity and drag plus buoyancy are 581 equal. Flocs are very likely to be captured by settling if their terminal velocity is higher than the 582 capture velocity of the settling tube.

583 The terminal settling velocity for flocs was defined by Adachi and Tanaka (1997) as:

584
$$v_t = \frac{g d_{clay}^2}{18 \Phi v_{H_2 O}} \frac{\rho_{clay} - \rho_{H_2 O}}{\rho_{H_2 O}} \left(\frac{d}{d_{clay}}\right)^{D_{fractal} - 1}$$
(8)

585 Where Φ is the shape factor for drag on flocs, d is the diameter of floc, v_{H_20} is the kinematic

586 viscosity of water, ρ_{clay} is the density of primary clay particles, ρ_{H_2O} is the density of water,

587 d_{clay} is the diameter of the primary particles, d is the floc diameter, and $D_{fractal}$ is the 3-D

fractal dimension of flocs. The shape factor accounts for the adjustment of the coefficient of drag
for non-spherical geometry and has a fractional value of 45/24 (Adelman et al., 2013).

- 590 Li and Ganczarczyk (1989) calculated the fractal dimensions of the alum aggregates based on the
- 591 reported data of settling tests and size-density relations. The fractal dimension from Boadway's
- 592 (1978) data is calculated to be around 2.3 and the one from Tambo and Watanabe (1979) is
- 593 between 1.59-1.97.
- 594 Figure 9 shows settling velocities predicted by equation (8) using a fractal dimension of 2.3.



Figure 9. Terminal velocity versus floc diameter.

597 In the experimental tube settler the removal efficiency of slow settling flocs of a specific size can
598 be quantified by pC*, which is defined by the following equation,

599
$$pC^* = -\log_{10}\left(\frac{C_{effluent}}{C_{influent}}\right)$$
 (9)

600 Where,
$$\frac{C_{effluent}}{C_{influent}} = 1 - \frac{v_t}{v_{capture}}$$
 (10)

 $C_{effluent}$ is the fraction of the effluent turbidity caused by the floc size class,

 $C_{influent}$ is the fraction of the influent turbidity caused by the floc size class,

$$v_{capture}$$
 is the capture velocity of tube settler.

604 Use of the ratio of terminal settling velocity and capture velocity to quantify removal of a floc 605 size class assumes that flocculation of particles does not occur in the tube settler. Figure 10 606 shows the expected pC* as a function of floc size, based on the predicted tube settler 607 performance for the ratio of terminal velocity to capture velocity.



608

609

Figure 10. pC* versus floc size.



611 0.21 mm/s, which means the removal efficiency of those flocs is expected to be 100%.

612 **2.6 Results**

613 2.6.1 Validation of image analysis method

614 The clay particle used for the test was kaolinite. Kaolinite particles have a reported diameter

for ranging from 0.2 μ m to 12 μ m (Aroke et al., 2013). The mean volume diameter of a 100 mg/L

616 kaolinite suspension (the turbidity was 68 NTU) was measured by a Mastersizer 2000 as 7.28

617 µm (Wei et al., 2015). Since the size of one pixel is close to the wavelength of visible light

618 (approximately 400-700 nm) (Pal and Pal, 2001), the diffraction of light can result in airy disks

- around particles in images and errors in particle size measurements.
- 620 Therefore, sizes of standardized particles were tested to determine the error caused by light
- 621 diffraction in the camera setup. More than 300 images of the suspensions of dark blue
- 622 polystyrene particles with nominal size of 3.0 μm (Sigma-Aldaich, Switzerland) were captured.
- Figure 11 is an example image of the standardized micro particles taken by the camera setup.



625 Figure 11. Image of standardized 3 μm polystyrene particles taken by the camera setup.

The images were then processed using the image analysis tool. The manufacturer determined the
diameter of the standardized micro particles using a Coulter multisizer II. As is shown in Table
3, the average particle size measured by the image system was greater than the values obtained
by the manufacture by 2.6 μm, which was consistent with the estimated diameter of the airy disk.

630

Table 3. Mean and standard deviation for 3.0 µm standardized particles

3.0 µm particles	Coulter multisizer (µm)	Image analysis (µm)
Mean	2.83	5.45
Standard deviation	0.07	1.09

The image analysis method was then used to measure the diameter of clay particles at different turbidities in the absence of coagulant, with the apparatus configured as shown in Figure 12.
Based on the calibration, a correction of 2.6 µm was subtracted from the mean diameter measured by the image analysis software. Calibration of particles of larger sizes can be applied in future study to verify the accuracy of the correction value of 2.6 µm.





637 Figure 12. Schematic of experimental set up for image analysis method verification.

The average measured diameter of the test clay particles was $7.7\pm3.8 \,\mu\text{m}$ after correction for the airy disk, which was within the reported size range for kaolinite and was quite close to the mean diameter of 7.28 μ m measured by Wei et al. (2015). Figure 13 illustrates the average measured

641 clay diameters at different turbidities.







653
$$\rho_{clay}$$
 is clay density and has a value of 2.5 g/cm³,

654 $1.73 \frac{mg}{L \cdot NTU}$ was measured in the Cornell Environmental Engineering laboratory by Casey

- 655 Garland (personal communication, June 13, 2015). This is similar to the value of $1.5 \frac{mg}{L \cdot NTU}$
- obtained by Wei et al. (2015).
- 657 The depth of field was calculated to be $500 \pm 90 \,\mu\text{m}$ for a range of influent turbidities. For flocs
- smaller than this depth of field, it is likely that the entire floc will be in focus.
- Figure 14 shows that there was a linear relationship between turbidity and the number
- 660 concentration of clay particles based on the depth of field calculated previously.



Figure 14. Number of clay particles per sample volume versus turbidity.

662

A linear fit with a zero intercept was obtained by calculating the average slope between each data point and the origin. The slope of the linear fit was $1.9E6 \pm 2.2E5\frac{1}{L\cdot NTU}$. Thus there were 1.9 million clay particles per $L \cdot NTU$. A $L \cdot NTU$ is equivalent to 1.73 mg of clay and given the density of clay is equivalent to a clay volume of 0.68 µL. Thus the average volume of the clay particles was 360 µm³ which yields an equivalent diameter of 8.8 µm. This is the volume weighted average diameter of the clay particles and thus gives a slightly larger diameter then the count weighted average diameter of 7.7±3.8 µm.

670 **2.6.2 Effect of coagulant dose**

671 Image analysis was performed on settled water along with measurement of effluent turbidity.

Figure 15 shows pC* values over a range of PACl doses. A pC* value of 1 indicates 90%

673 removal efficiency a pC* of 2 indicates 99%, and so on. The PACl doses applied to a 50 NTU

674 raw water were 0.53 mg/L, 1.06 mg/L, 1.59 mg/L, 2.11 mg/L and 2.65 mg/L as aluminum. As is

shown in Figure 15, pC* increased when PACI dose increased and there was a linear relation

between pC* and the logarithm of PACl dose.





Figure 15. pC* versus PACL dose (mg/L as Al).

The result in Figure 15 agree with the flocculation model created by Swetland et al. (2014).
These investigators observed a linear relationship between pC* and the logarithm of colloid
surface coverage by coagulant (which is proportional to coagulant dose at low doses). The slope
in Figure 15 was 1.1 and is close to the slope of 1 for the model indicated by Swetland et al.
(2014).

684 Figure 16 illustrates the distribution of floc number concentration allocated to different bin sizes.



Figure 16. Floc size distributions of settled water according to different bin sizes (PACl dose = 0.53 mg/L as Al).

688 The horizontal axis in Figure 16 is the spherical-equivalent floc diameter while the vertical axis 689 is the total number of flocs per sample volume per bin size within each floc size range. For each 690 set of data, bin size was varied in a power law relation to a base. For instance, when the base is 691 selected to be 1.3, the size of the first bin would be 1.3 µm and the first bin is defined by a lower bound of 0 µm and an upper bound of 1.3 µm, the second bin size is 1.3^2 µm (=1.7 µm) and the 692 lower and upper bounds are 1.3 μ m and 3.0 μ m. The third bin is 1.3³ μ m (=2.2 μ m) and its lower 693 694 and upper bounds are 3.0 μ m and 5.2 μ m, and so on. The median value of each bin is considered 695 as the mean diameter of flocs for that size range. The number of flocs within each bin was then 696 counted by the LabVIEW histogram function.

697 For alternative bases, there were only slight changes in the shape of the distribution curve and

the area under the curve. This result indicates that the specification of bin size over the range

tested had little impact on the particle size distribution curve.

Figure 17 shows a log-log plot of the floc number concentration versus size. The bin sizes for
each base were determined as described above. The changes in bin sizes had little influence on
the log-log plot of floc size distribution.



703

Figure 17. Log-log plot of floc size distributions of settled water according to different bin sizes (PACl dose = 0.53 mg/L as Al).

Figure 18 indicates that the number concentration of flocs in the settled water decreased with the

707 increase in PACl dose. As is shown in Figure 9, the largest floc predicted to escape the tube

settler at a capture velocity of 0.21 mm/s is 120 μ m. As seen in Figure 18, the maximum floc

size observed in the settled water was less than $120 \,\mu$ m, which is in agreement with the model.



711 Figure 18. Floc size distributions of settled water at different PACl dose (mg/L as Al).

The data in Figure 19 suggests that the number concentration of flocs in the settled water

713 decreases as a function of coagulant dose. Both an exponential and power law provided a good

fit to the data in Figure 19. The fits to the data are shown in Table 4.





716 Figure 19. Floc number concentration in the settled water versus PACl dose (mg/L as Al).

717 At low coagulant doses, pC* had a linear relationship with the logarithm of PACl dose,

718 indicating that turbidity and coagulant dose followed a power law relation (see Figure 15). The

sum of squared errors of prediction (SSE) of exponential fit was lower than that of power law fit

(see Table 4). However, it was difficult to conclude which regression better fit the data in Figure

19 because both the r squared values shown in Table 4 were quite high. Further studies over a

vider range of coagulant doses (coagulant doses less than 1 mg/L as Al) should be conducted to

see how floc number concentration is reduced as a function of coagulant dose.

724

Table 4. Exponential fit and power law fit in Figure 19.

Trend line option	Trend line equation	r ²	SSE
Exponential fit	$y = 2 \times 10^7 e^{-1.6x} \frac{1}{L}$	1.00	2.3×10 ¹²
Power law fit	$y = 3 \times 10^6 x^{-2.1} \frac{1}{L}$	0.96	4.1×10 ¹²

728

729 **2.6.3 Comparison between flocculated water and settled water**

In Figure 20, floc size distributions are compared between flocculated water and settled water to
evaluate the performance of the tube settler. Flocculated water was sampled after the flocculator,
while settled water was sampled after the tube settler.



733



736 The results confirm that sedimentation does little to remove particles below the capture velocity

737 of the sedimentation tank. One concern with the results shown in Figure 20 is that the

738 concentration of small flocs (less than 5 μ m) in the settled water was higher than that in the 739 flocculated water. The inner diameter of the connecting tube between the tube settler and the 740 turbidimeter was constrained by the 0.95 cm exit port diameter of the tube settler (see Figure 22 741 in Appendix. C), thus the velocity gradient inside the connecting tube was 87/s, 24% higher than 742 that inside the flocculator. The higher shear inside the connecting tube may break big flocs into 743 small ones. However, preferential production of floc fragments smaller than 5 µm would not be 744 expected. Another explanation for the observed increase in small flocs might be overlapping of 745 flocs in the depth of field within the image volume. When there is a large floc in the image, small 746 flocs behind or in front would not be detected by image analysis. The flocculated water has more 747 large flocs (diameter greater than 70 µm) than the settled water. Therefore, the number of small 748 flocs in flocculated water is more likely to be under-estimated due to the image occlusion caused 749 by big flocs. The number concentration of flocs for each bin size could possibly be corrected for 750 occlusion by larger flocs to improve this analysis. The occluded volume would be obtained by 751 the area of the larger flocs multiplied by the calculated depth of field.

Figure 21 shows pC* for the three coagulant doses as a function of floc sizes and the expected
pC* based on predicted tube settler performance for the ratio of terminal velocity to capture
velocity.



756

Figure 21. pC* value versus floc size.

The observed concentration changes between settled water and flocculated water were negligible
(pC* of less than 0.2) except for particles that approach the capture velocity of the sedimentation
tank.

760 **2.7 Conclusions**

This paper presents an effective way to employ digital image analysis to continuously count and size flocs in a flow-through-cell. Out-of focus particles are automatically identified and excluded thus improving the accuracy of the results of floc size measurement. The constraints for floc size measurements are the field of view, the depth of field and the airy patterns caused by the objective lens. The apparatus could measure particle sizes ranging from around 2.6 µm to more than 300 µm. The error in measuring particle sizes caused by airy disk (light diffraction) was measured by testing particles of known diameter. The influence of airy disk accounted for a
correction of 2.6 µm, which was consistent with the estimated diameter of the airy disk.

The average particle diameter of the test suspension of kaolinite clay was measured to be 770 $7.7\pm3.8 \,\mu\text{m}$ and a linear relationship was obtained between turbidity and the concentration of 771 clay particles determined by imaging.

Size distribution of flocs could be plotted in varying bin sizes, when the bin sizes increased with particle size following a power law. Since there are fewer large flocs, the bin size was kept proportional to the bin mean diameter to ensure that sufficient flocs were in the large bins to obtain a statistically meaningful particle count in each bin. Thus, varying the bin size with floc diameter can better reveal the shape of the size distribution curve. The shape and the area under the size distribution curves were independent of the bases used to set bin size.

For settled water, as was expected, floc number concentrations decreased when the PACl dose increased. pC* had a linear relationship with the logarithm of PACl dose. The maximum floc size observed in the effluent was less than 120 μ m, which was in accordance with the value predicted by a model for the capture velocity of the experimental tube settler. Image occlusion caused by overlapping flocs may result in the underestimation of the number concentration of small flocs in flocculated water.

Image analysis of flocculated water could be used to predict particle counts after sedimentation.
This has the potential to be used to improve performance of water treatment plants especially
during raw water quality changes.

787 **2.8 Future work**

The value of α_t and β_t were determined based on looking at a range of computed α and β from large sets of floc images. The determination of α_t and β_t may be influenced by the floc image sample size (the number of floc images taken from the sample cell). In addition, the sample size may also affect the particle size distribution curve during flocculation. The study of the effect of variances in sample sizes on particle size distribution could improve the analysis.

793 At low coagulant doses, pC* had a linear relationship with the logarithm of PACl dose, 794 indicating that turbidity and coagulant dose followed a power law relation. However, the 795 relationship between floc concentration in the settled water and coagulant dose was less 796 apparent. Both exponential and power law regressions fit the data well. In future experiments, a 797 wider range of coagulant doses (coagulant doses less than 1 mg/L as Al) could be applied to the 798 flocculator to check how floc number concentration is reduced as a function of coagulant dose. 799 The floc size distribution may follow a power law relation, $N(d) \sim d^{-p}$, where N(d) is the number 800 of flocs per sample volume within the diameter range of d to $d+\Delta d$. The slope (-p) of the particle 801 size distribution can vary depending on coagulation mechanisms, such as Brownian motion, fluid 802 shear and differential sedimentation. The observations of particle size distributions in natural 803 water sources indicate that the collision mechanisms of small (less than 2 µm) and median 804 particles (2~60 µm) are dominated by Brownian motion and fluid shear respectively, while big 805 flocs (greater than 60 µm) might be formed as a result of differential sedimentation (Li, et al., 806 2004). Thus, the change of the slopes in the floc size distribution may indicate different 807 flocculation mechanisms for different size of particles. Future study on the slope changes may

808 improve our understanding of the interaction mechanisms between colloids as well as predict the
809 evolution of floc size distribution under different operation conditions.

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885	

APPENDIX

887 A. Calculation of \overline{G} , Q, L

Under laminar flow conditions (i.e. when the Reynolds number, $Re \le 2100$), the velocity of fluid at a radial distance r from the pipe axis, through a straight pipe with a circular cross section, can be expressed by equation (12) (Gregory, 1981),

891
$$v_{r=} = v_0 \left(1 - \frac{r^2}{R^2} \right)$$
 (12)

892 where, v_r is the velocity of fluid at a radial distance r from the pipe axis,

- 893 v_0 is the maximum velocity in the fluid (the axis velocity),
- 894 r is the radial distance from the pipe axis,
- 895 *R* is the radius of the pipe.

896 The average velocity of the fluid can be obtained by integrating Equation (12),

897
$$\overline{\nu} = \frac{\int_0^R v_0 \left(1 - \frac{r^2}{R^2}\right) 2\pi r dr}{\int_0^R 2\pi r dr} = \frac{v_0}{2}$$
(13)

898 Gregory (1981) argued that the velocity gradient G at a radial distance from the pipe axis could899 be defined by differentiation of equation (12),

900
$$G = \frac{dv_r}{dr} = 2v_0 r/R^2$$
 (14)

901 G increases from zero at the pipe axis to a maximum value at the wall.

902 The average velocity gradient in the pipe can then be calculated as,

903
$$\overline{G} = \frac{2v_0}{R^2} \frac{\int_0^R 2\pi r^2 dr}{\int_0^R 2\pi r dr} = \frac{4v_0}{3R}$$
(15)

904 Since
$$Q = \frac{v_0}{2} \pi R^2$$
, (16)

905
$$\overline{G_{Gregory}} = \frac{8}{3} \frac{Q}{\pi R^3} = \frac{64}{3} \frac{Q}{\pi D^3}$$
 (17)

Camp and Stein (1943) suggested that the average velocity gradient could be obtained from the

907 power input to the volume of the pipe:

908
$$\overline{G} = \sqrt{\frac{P}{\mu V}}$$
(18)

- 909 Where, P is the power input of the system,
- 910 μ is the dynamic viscosity of the fluid,

912 The power input of the system could be expressed as the product of the flow rate and the

913 pressure drop across the tube,

914
$$P = Q\Delta p \tag{19}$$

915 For a long cylindrical pipe, the Hagen-Poiseuille law (Pfitzner, 1976) leads to the calculation of

916 the pressure drop across the pipe.

917
$$\Delta p = \frac{128Q\mu L_{tube}}{\pi D^4} \tag{20}$$

918 Where, L_{tube} is the length of the flocculator tube.

919 Combing equation (18), (19), (20) and considering $V = \pi R^2 L$, the Camp Stein expression of 920 average G can be represented by:

921
$$\overline{G_{CampStein}} = \frac{16\sqrt{2}Q}{\pi D^3}$$
(21)

922 For the experimental conditions, the average velocity gradient calculated by Camp Stein923 (70.57/s) is 6% higher than that from Gregory.

The energy dissipation rate used in this research was: $\varepsilon = 5 \frac{mW}{kg}$. The velocity gradient can be 924 calculated by equation (1) and (2) and thus the estimated average G would be $70.6s^{-1}$. A range 925 of Q values were substituted into equation (2) to obtain the desired experimental velocity 926 gradient. At $Q_{plant} = 215 \frac{mL}{min}$, G = 70.6 s⁻¹. When G θ is greater than 20,000, it is expected that 927 928 there would be successful flocculation in the flocculator (Camp and Stein, 1943). For safety, a 929 $G\theta$ value of 21,000 was selected. Hence, the residence time and the length of the tube could be separately calculated by equation (22) and equation (23), where A_{Tube} is the cross sectional area 930 931 of the tube.

$$932 \quad \theta = \frac{21000}{G} \tag{22}$$

933
$$L_{tube} = \theta \frac{Q}{A_{tube}}$$
(23)

The resulting residence time of the flocculator was 300 s and the length of the tube was 49 ft.

935 **B. Flow rate, coagulant dose and influent turbidity**

The constraints of the flow rate through the apparatus were the minimum flow rates required for the turbidity meter and the maximum rate for the flow cell. A flow rate of 3.58 ml/s through the apparatus met the minimum 1.67 ml/s requirement for the turbidity meter.

939 The flow rate needed for the coagulant solution was calculated by the law of conservation of940 mass.

941
$$Q_{Al} = Q_{plant} \times \frac{c_{plant}}{c_{Al}}$$
(24)

942 Where, Q_{Al} is the flow rate of coagulant solution,

943
$$Q_{plant}$$
 is the flow rate through the flocculator,

- 944 C_{plant} is the Al dose within the flocculator,
- 945 C_{Al} is the Al concentration of coagulant stock.
- 946 Equation (25) was used to calculate the concentration of clay added to water, the value of
- 947 $1.73 \frac{mg}{L \cdot NTU}$ was measured in the lab by Casey Garland (personal communication, June 13, 2015).

948
$$C_{clay} = 1.73 \frac{mg}{L \cdot NTU} \cdot Target NTU$$
 (25)

949 C. Tube settler

- 950 The 1.37 m (4.5 ft) tube settler (whose inner cross sectional dimensions are 2.22 cm×2.22 cm)
- has an entry port diameter of 1.3 cm (½ in) near the bottom and an exit port diameter of 0.95 cm

- 952 (3/8 in) near the top, as is shown in Figure 22. A 0.32 cm (1/8 in) diameter tap, is located on the
- 953 end near to the bottom. The tap is used as a drain to remove flocs.



961
$$v_{capture} = \frac{v_{up} \cdot s}{L_{tube \ settler} \cdot \cos \theta \cdot \sin \theta + s}$$
 (27)

962 **D. Number concentration of primary particles**

963 The number of primary particles in each floc was calculated by the following equation,

964
$$n_i = \left(\frac{d}{d_{clay}}\right)^{D_{fractal}}$$
 (28)

965 The total number of primary particles within sample volume can be obtained from,

966
$$n_{total} = \sum_{i=1}^{k} n_i \tag{29}$$

967 Where n_i is the number of primary particles in floc *i*,

968 k is the total number of flocs and $D_{fractal}$ was assumed to have the value of 2.3 reported 969 by Li and Ganczarczyk (1989).

970 Figure 23 shows the estimated number of primary particles per sample volume in the effluent for

971 each PACl dose (mg/L as Al). The straight line in the graph is the zero-intercept linear fit

972 determined in Figure 14, which was used to predict the number of clay particles per sample

volume at a given effluent turbidity. Turbidity of flocculated water was measured and turned out

974 to be almost the same as the turbidity of raw clay water (without coagulants). Hence, the

975 expectation was that turbidity would be related to the concentration of primary particles,

976 meaning data points in Figure 23 should fit the relationship determined for unflocculated clay

977 suspensions.

978 Turbidity~n_{total}

(30)



Figure 23. Estimated number of primary particles in the effluent at different aluminum doses based on an assumed fractal dimension of 2.3. Solid line is fit of number of particles per NTU based on Figure 14.



The relationship between turbidity and primary particle number concentrations in flocs was less
apparent, perhaps due to the assumption of an incorrect fractal dimension value. Further studies
should be conducted on the determination of 3-D fractal dimension from 2-D floc images. This
relationship will enable prediction of turbidity based on image analysis.
Figure 24 indicates that the number concentration of primary particles in the settled water

decreased with the increase in PACl dose. Flocs ranging from 20 to 50 µm in diameter accountedfor the greatest proportion of the primary particle concentration.



997

Figure 24. Primary particle distribution in the settled water at different PACL dose (mg/L as Al).

1000 As illustrated in Figure 10, the experimental tube settler can achieve 100% removal of flocs

1001 greater than 120 μ m while flocs around 68 μ m would be expected to be removed with 50%

1002 efficiency in the tube settler. Thus, big flocs (greater than 70 µm) occupied a small proportion of

1003 the mass due to their high removal efficiency. Flocs less than 10 µm account for less mass

1004 perhaps as a consequence of flocculation.

1005 Primary particle distributions were compared between flocculated water and settled settled water

1006 to evaluate the performance of the tube settler in Figure 25. The concern in Figure 25 is the same

1007 as the one in Figure 20, as was mentioned in part 2.6.3.

1008



Figure 25. Primary particle distribution of flocculated water and settled water at different
 PACl doses (mg/L as Al). (TS designates tube settler.)

1012 E. Fractal dimension

1013 Fractal dimension is a crucial parameter in determining the floc shape, density, porosity, and

1014 settling velocity, as well as their kinematic behaviors, such as particle aggregation and breakup.

1015 Numerous ways have been suggested to calculate the 3D fractal dimension. These ways include 1016 direct methods, such as a box-counting method (Vahedi and Gorczyca, 2011), and indirect 1017 methods, like a free settling test. One of the direct methods to determine the three-dimensional 1018 fractal dimension will be discussed here. This method is to relate the number of primary particles 1019 to the floc diameter (Meakin, 1998),

$$1020 d = d_{clay} n_i^{\frac{1}{D_{fractal}}} (31)$$

1021 The number of primary clay particles in 3D dimension (n_i) could be estimated from the number 1022 of clay particles in the 2D image (n_0) , based on the assumption that the flocs were spherical. 1023 Thus, the total number of primary particles in 3D dimension would be

1024
$$n_i = \frac{4}{3}\pi \left(\sqrt{\frac{n_0}{\pi}}\right)^3$$
 (32)

1025 Where, n_0 is the number of primary particles counted in a 2D image.

1026 The fractal dimension could be then calculated by the power law fitting of equation (31).

1027 Figure 26 shows several sample images of flocs. In terms of equation (32), the number of

1028 primary particles was counted to determine the fractal dimension of flocs. Using the graphs in

1029 Figure 26 for illustration, there were approximately twenty clay particles in the first floc. The

1030 second and third floc images have 22 and 15 clays each.



1032

Figure 26. Sample images of flocs.

- 1033 After regression analysis, the power in Figure 27 was calculated as 0.53. The 3D fractal
- 1034 dimension was calculated according to equation (31) and the result was approximately 1.9,
- 1035 which was within the range of 1.6~2.3 indicated by Li and Ganczarcayk's (1989) result. The
- 1036 constraint of this method is that it could only count the clay number in flocs smaller than $50 \,\mu m$.
- 1037 For flocs larger than 50 μ m, the aggregates were too densely packed to count.





1039 Figure 27. The regression of floc diameter vs. estimated primary particle numbers.

1040 Maggi (2007) proposed another way to calculate perimeter-based fractal dimension. It is

1041 calculated to compare with the 3D fractal dimension attained by the power law fit of equation

1042 (31).

1043 In Maggi's theory, the 2D fractal dimension is defined as

$$1044 D_p = 2 \frac{\log P_{pixel}}{\log A_{pixel}} (33)$$

1045 Where P_{pixel} is the perimeter of the floc while A is the projected area of the floc. Both P_{pixel} and

- 1046 A_{pixel} are in in units of pixels. D_p ranges from 1~2. The 3D volume fractal dimension D_v can
- 1047 then be derived from D_p when D_p is smaller than 2.
1048
$$D_{\nu} = \sqrt{\frac{a(x)}{D_p - b(x)}}$$
 (34)

1049 Where $x = \frac{d}{l_{pixel}}$, is the dimensionless floc size, *d* is the floc diameter and l_{pixel} is the pixel size.

1050 a(x) and b(x) are used to take the resolution into account and can be calculated from the

1051 following equations,

1052 a(x) = 9[z(x) - b(x)] (35)

1053
$$b(x) = \frac{2[k(x)^2] - 9z(x)}{[k(x)^2] - 9}$$
 (36)

1054 Where
$$k(x) = z(x)[z(x) - 1] + 1$$
 (37)

1055
$$z(x) = \frac{\log(4x-4)}{\log x}$$
 (38)

1056 Figure 28 shows the D_{v} calculated by equation (34).



Figure 28. Volume fractal dimension of flocs.

1059The average fractal dimension value was 2.51 ± 0.22 . The fractal dimension values are presented1060in the double-logarithmic plot graph. The values decreased with increasing the dimensionless1061size d/d_0 , indicating that flocs appeared less dense and much more loosely clustered as the floc1062diameter increased.

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