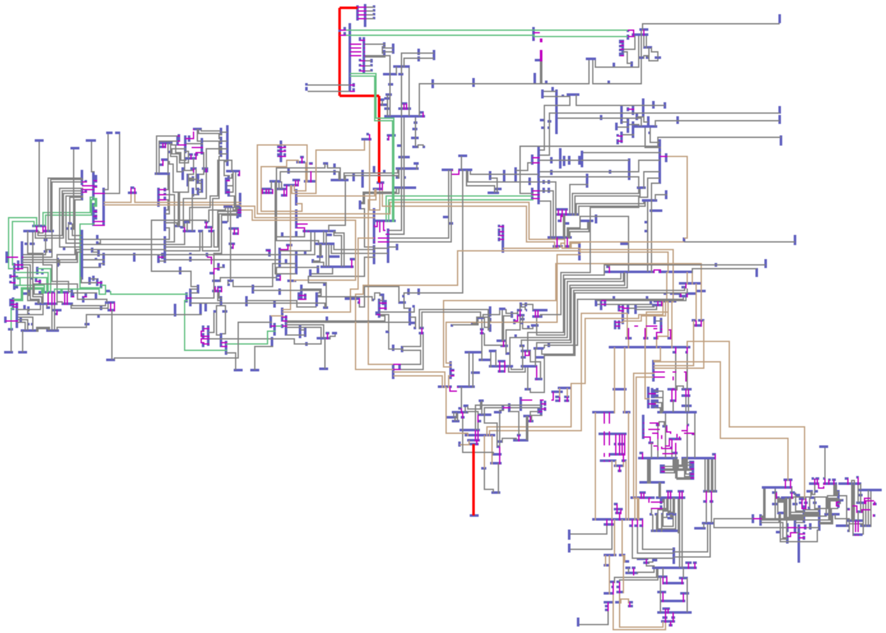
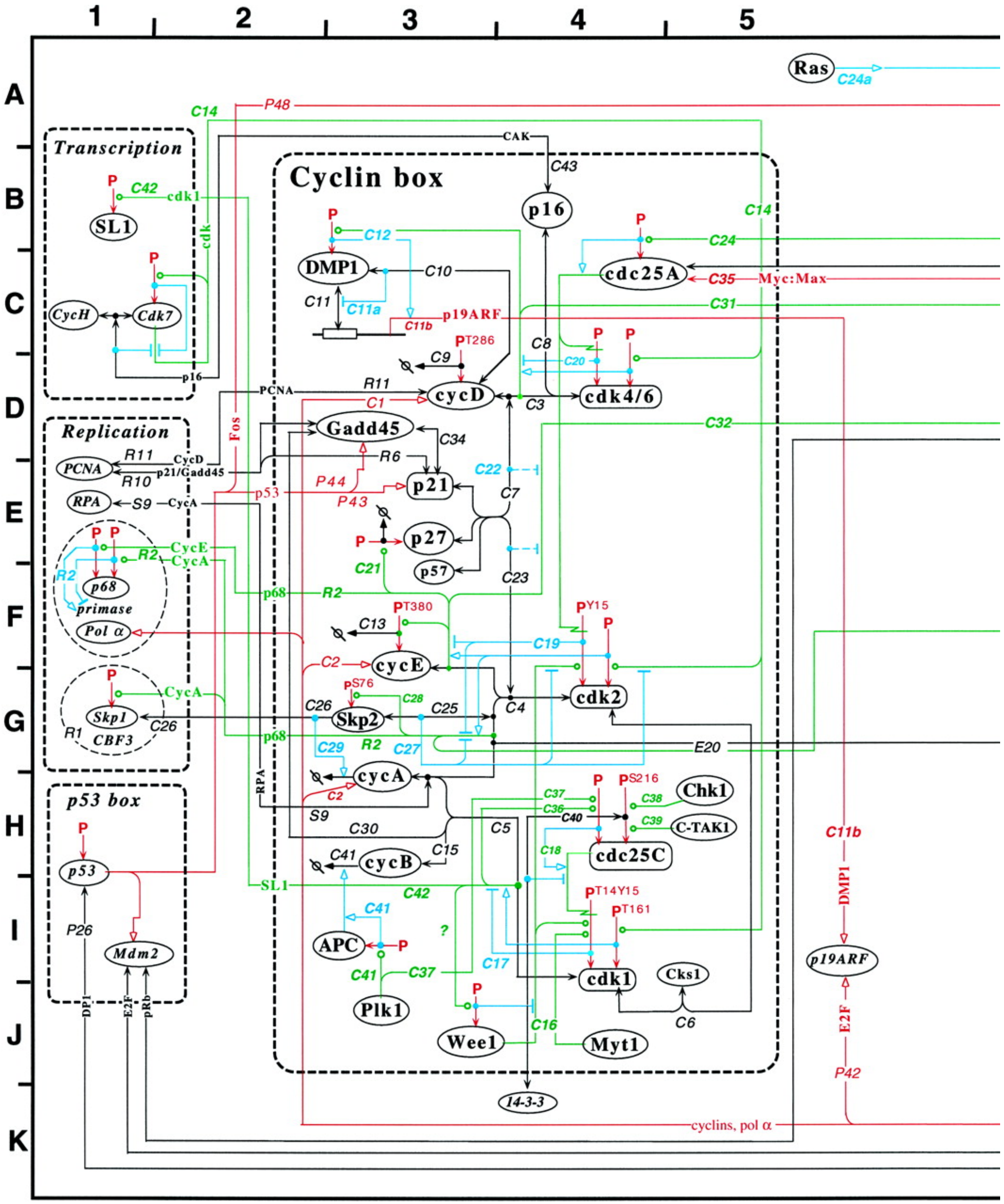
Complex systems as networks

Power grid



(Courtesy of J. Thorp)



Cell cycle

(Courtesy of K. Kohn)

Why networks are hard

1. **Structural complexity:** wiring diagram could be an intricate tangle
2. **Network evolution:** wiring diagram could change over time (World Wide Web)
3. **Connection diversity:** links could have different signs, weights, and directions (synapses in nervous system)
4. **Dynamical complexity:** nodes could be nonlinear dynamical systems (genes, power plants)
5. **Node diversity:** there could be many different types of node (cell cycle)
6. **Meta-complication:** the various complications can influence each other
   - Layout of power grid depends on how it has grown -- (2) affects (1).
   - Neurons that fire together, wire together (as in memory and learning) -- (4) affects (3).

Unifying principles?

“Small world” architecture is pervasive …

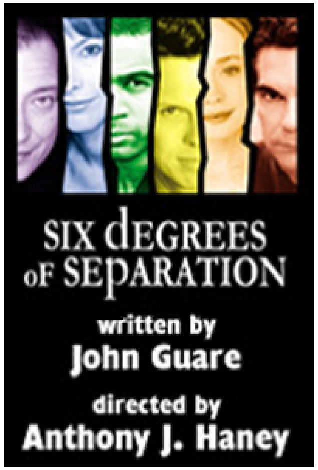
(Watts and Strogatz, *Nature* 1998)

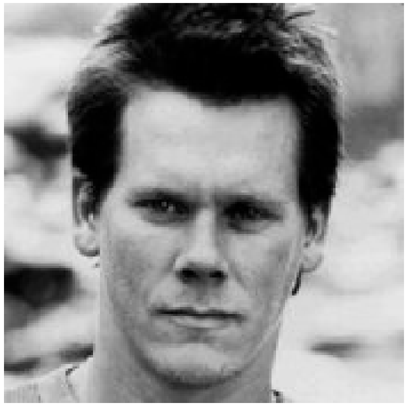
… and so is “scale-free” distribution of connectivity

(Barabasi and Albert, *Science* 1999)

The small-world problem

Stanley Milgram





Small world poses a math problem

The problem: How can a network be both very small and very clustered?

Real networks are “small worlds”

	L_{Actual}	L_{Random}	C_{Actual}	C_{Random}
Movie actors	3.65	2.99	0.79	0.00027
Power grid	18.7	12.4	0.080	0.005
C.elegans	2.65	2.25	0.28	0.05

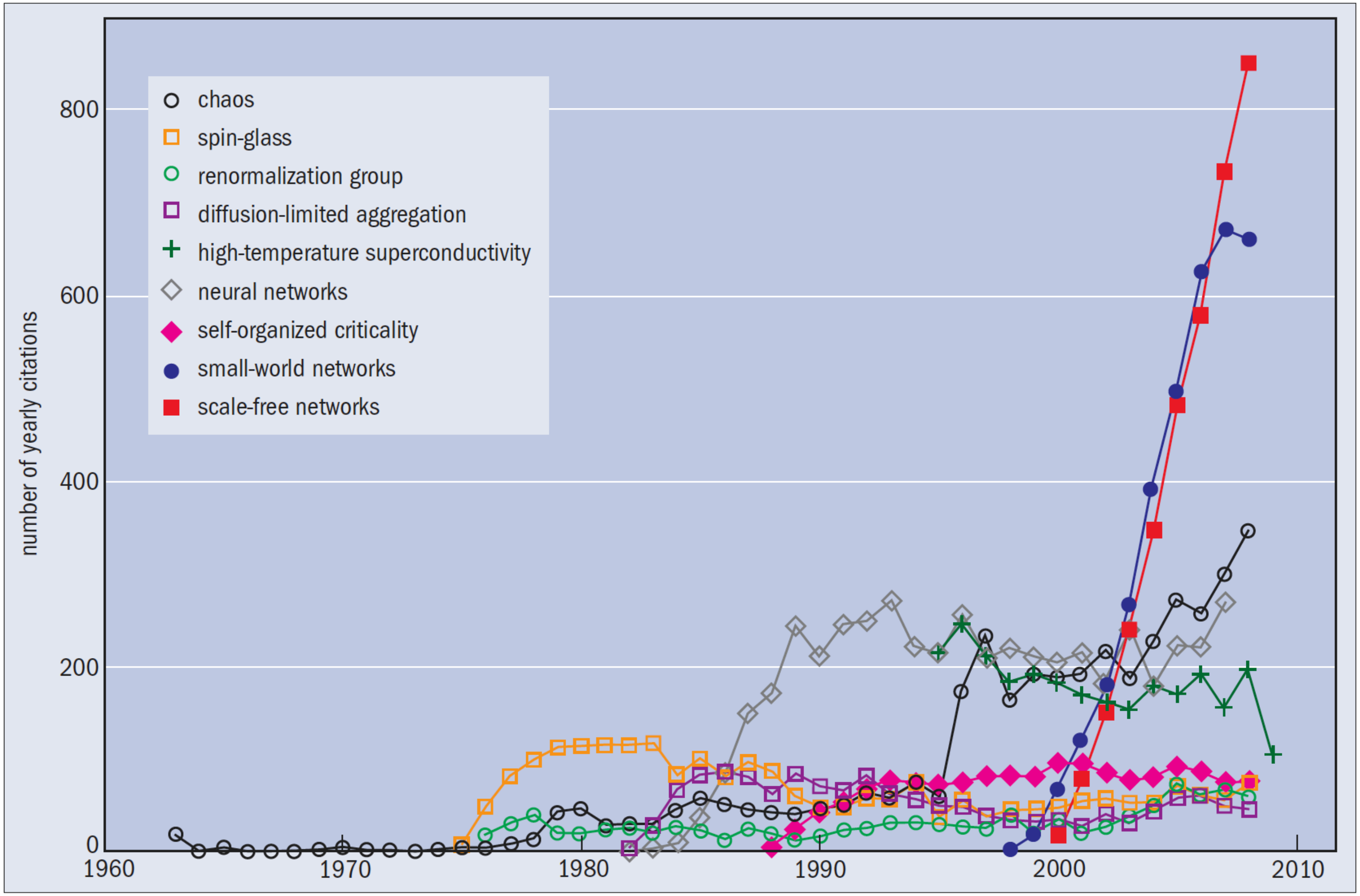
L = average degrees of separation; C = clustering coefficient

Functional implications

- Fast synchronization in small-world nets

  Relevance to epilepsy, cognition…

- Improved parallel computation
- Fast spreading (information, contagion)
- Finding a job ("strength of weak ties")



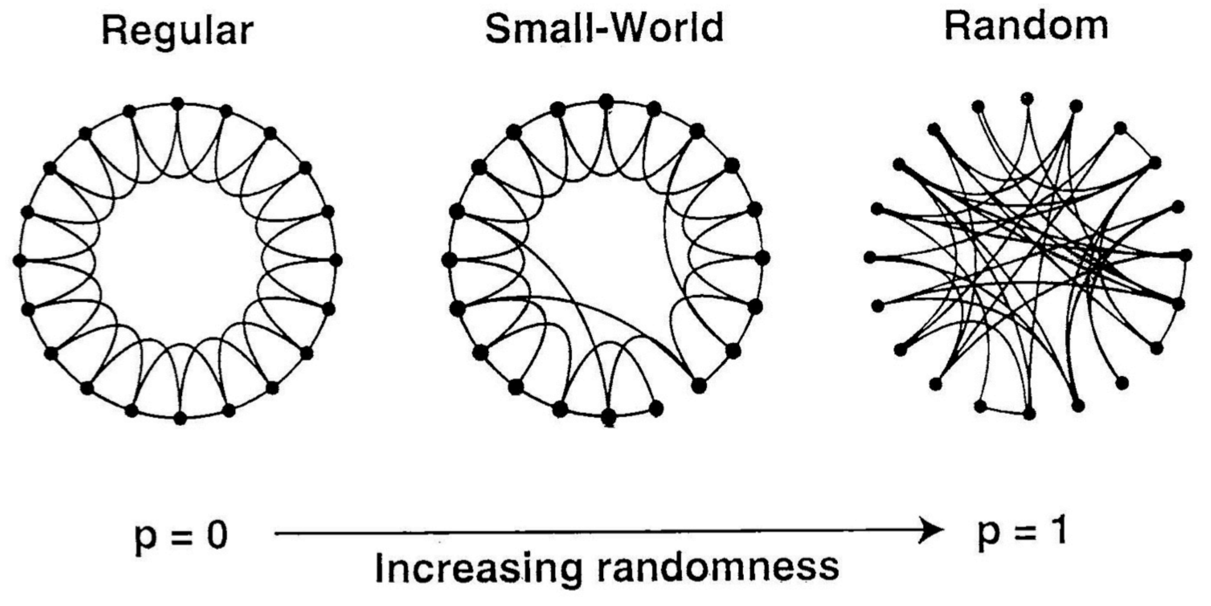
Networks take off This graph of the number of citations garnered by a handful of groundbreaking physics papers shows that even within this elite group, the popularity of papers on networks has been unprecedented.

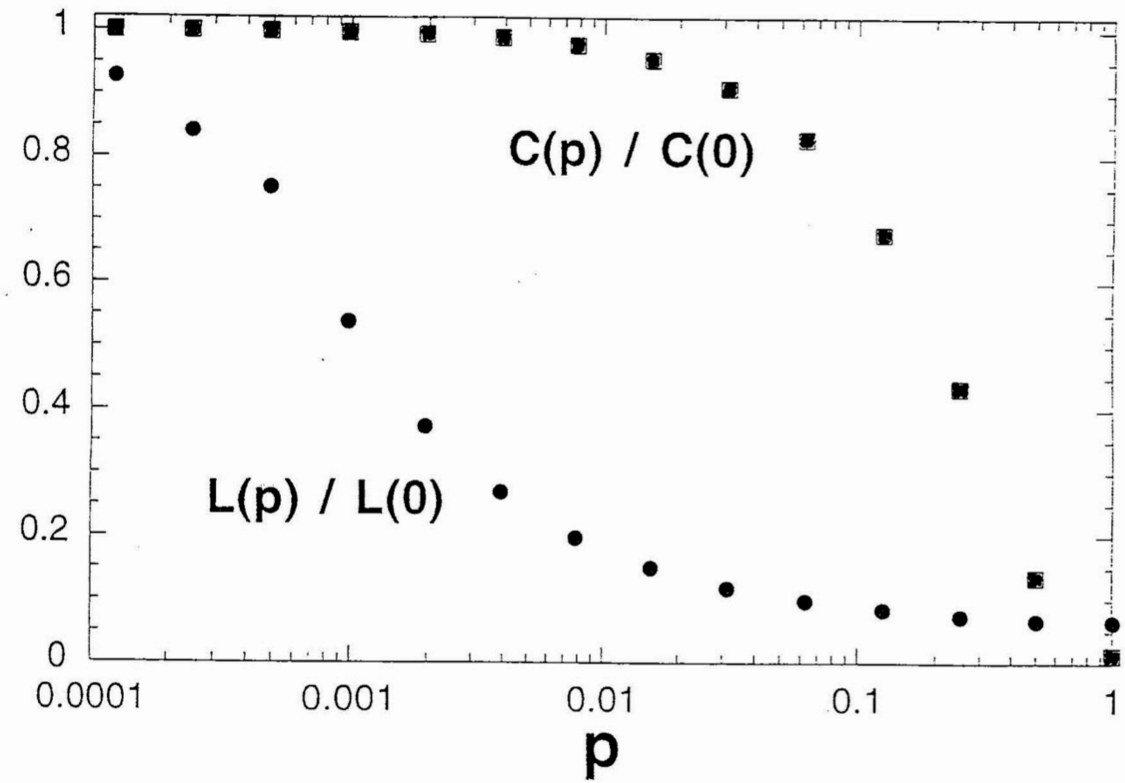
“The greatest challenge today, not just in cell biology and ecology but in all of science, is the accurate and complete description of complex systems.

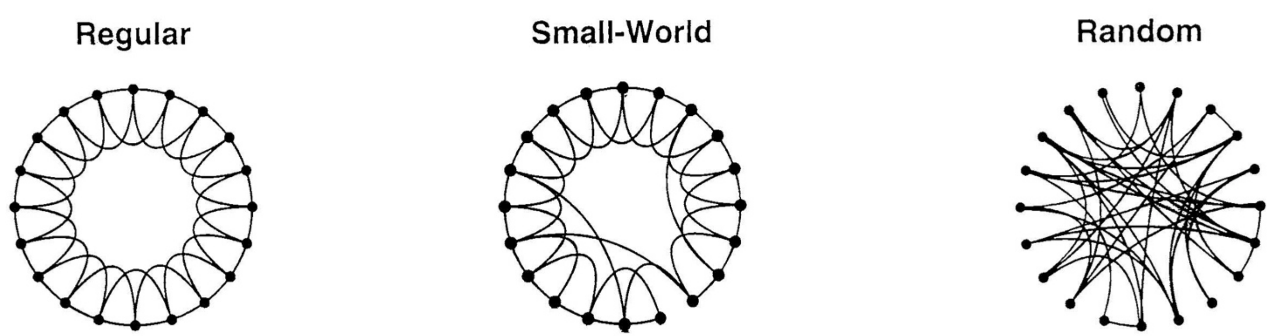
Scientists have broken down many kinds of systems. They think they know most of the elements and forces. The next task is to reassemble them, at least in mathematical models that capture the key properties of the entire ensembles.”

E.O. Wilson
Consilience (1998)

A “thought experiment”




C(p) / C(0)
L(p) / L(0)
1
0.8
0.6
0.4
0.2
0
0.0001
0.001
0.01
0.1
1
p


Regular
Small-World
Random

Degree distributions

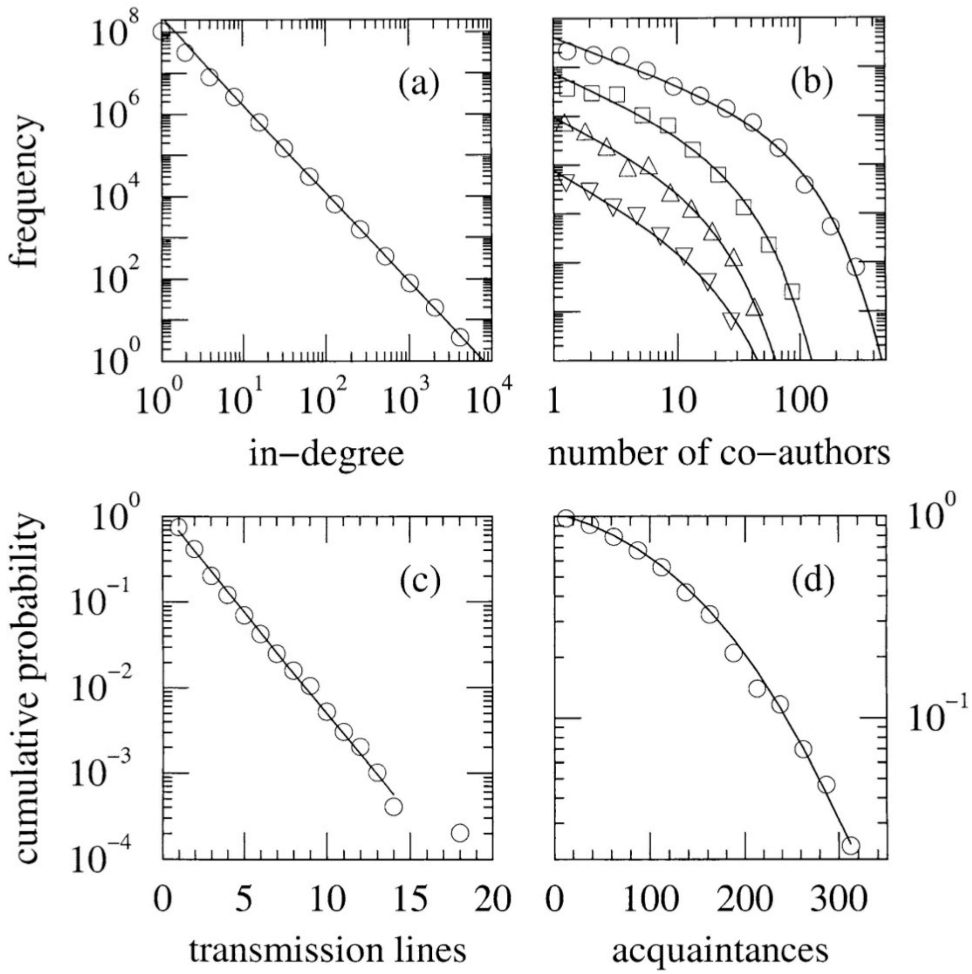
(some nodes are more connected than others)

Web pages

Power-law
= “scale-free”

Scientists

Truncated
power-law



Power grid

Exponential

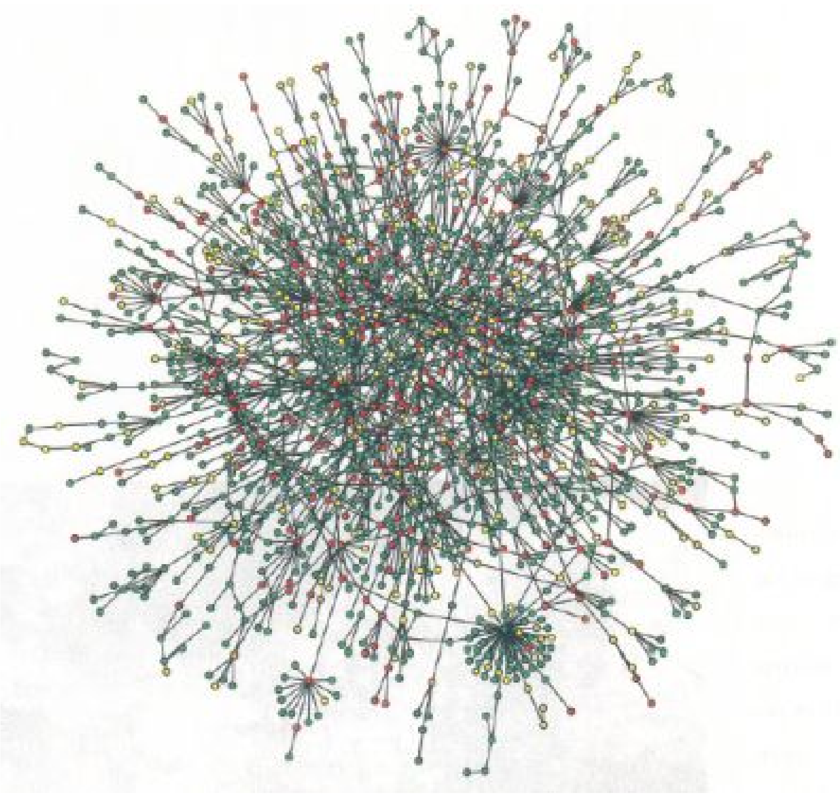
Mormons

Gaussian

Protein interactions (yeast)

- Hubs: Top 1% of proteins have > 15 links
- Deletion of a hub is lethal, 62% of time

- 93% of proteins have < 5 links
- Deletion of them is lethal only 21% of time



Jeong et al., *Nature* (2001)