Convection Flow in AguaClara Plate Settlers

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Abstract

The AguaClara project team develops a simple, low-cost water purification plant for developing global communities. One part of the AguaClara plant is a sedimentation tank. During this stage of the purification, large coagulated contaminant particles (”flocs”) collect and settle out of the water, leaving it cleaner than before. Recently, one problem has been noticed with the current design of the AguaClara sedimentation tank. If the water entering the tank is warmer than the water already within, the tank doesn’t perform as well as it should. The suspected reason for this is that the warm buoyant water rises rapidly to the top of the tank, taking the flocs with it, at a velocity that is too great for the tank to function correctly. The top of the tank consists of an array of angled closely packed parallel plates called plate settlers. My project here is to determine the flow pattern for a convective flow through the plate settlers, with the hope that this will provide a clearer picture of the poor sedimentation tank performance. A 2d analytical solution is determined for the flow velocity between two angled parallel plates when the flow is convective. Further topics are discussed as they relate the sedimentation tank performance, such as turbulence between opposing flows and shear flow instabilities.

1 Background on the AguaClara Sedimentation Tank

The AguaClara sedimentation process has several stages. First, the influent water is injected with a coagulant that causes particles to stick together, forming ”flocs”. The water then passes through a series of twists and turns in the ”flocculator”, which causes the flocs to bump into each other and form larger flocs. The flocculated water is then injected into the bottom of a very large trough-shaped tank called the sedimentation tank. The water then travels up through the tank, through a very thick suspended layer of flocs called a ”floc blanket”. The upward velocity during this portion of the process is very slow by design (on the order of 1 mm/s), so that flocs have time to settle out of the mixture before the water leaves. Larger flocs have larger sedimentation velocities, which means that they sink easier in water and thus settle more quickly than smaller flocs. This is why so much of the design is focused on forming big flocs from many smaller flocs. Figure 1 shows a side-view of the sedimentation tank from [1].

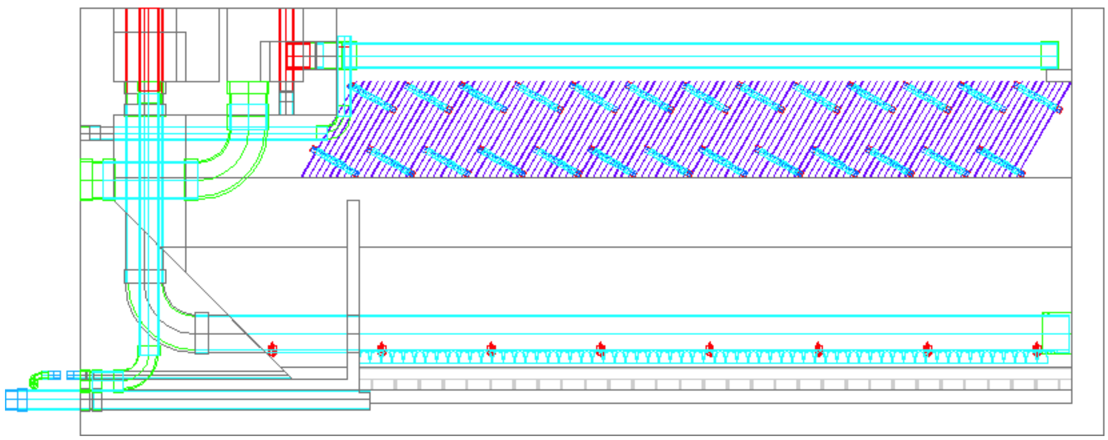

Figure 1: Technical drawing of the sedimentation tank

At the top of the sedimentation tank is a array of close packed plates positioned at an angle of 60° with respect to the horizontal. These are the plate settlers and they are the point of focus for this project. The reason that the plates are packed close together and are positioned at an angle is that this geometry increases the amount of flocs that will settle out. The water must travel the long distance between one end of the plate and the other, but in the time that it takes to do that, the floc will settle out if it descends only the short distance between two plates. Figure 2 illustrates the typical flow through the plate settlers.

In terms of dimensions, the sedimentation tank is about 4m long, 1m wide, and 1.5 m tall [1]. The spacing between settler plates is around 2.5 cm. Typically average upflow velocity is 1 mm/s.

All the math related to the sedimentation in the plate settlers is carried out in detail in [2]. One key detail is that the sedimentation velocity of the slowest floc that can be captured by the plate settlers is given by

$$V_c = \frac{V_\uparrow}{\frac{L}{D}\cos(\phi)\sin(\phi) + \sin^2(\phi)} \tag{1}$$

Where V_c is the capture velocity (the slowest sedimentation velocity that will settle), L is the length of the plates, D is the spacing between them, and ϕ is the angle of the plates with respect to horizontal.

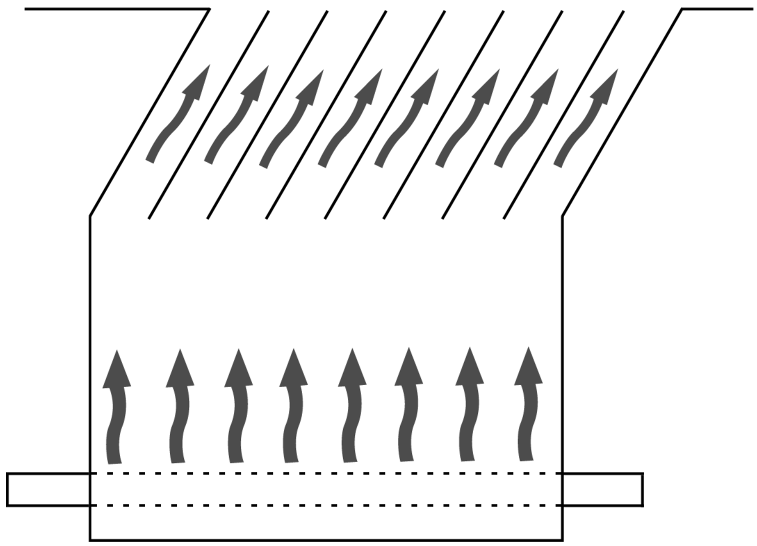

Figure 2: Typical flow through the plate settlers

Thus, if the vertical component of water velocity is increased, only the flocs with higher sedimentation velocity (and thus larger size) will be able to settle.

Recently, the AguaClara plants have been having poor performance during certain hours of the day. It is suspected that the cause of this problem is that during peak hours of the day, the pipes leading to an AguaClara plant are exposed to the sun. The sun heats up the water in the pipes, and when this water reaches the sedimentation tanks, it is warmer than the water already within the tank. Empirically, it has been observed that the influent water temperature increases by about 1°C every hour. This temperature difference (although small), is enough to drive convection in the tank, and these convective flow patterns prevent optimal performance of the sedimentation tank.

2 Past Research and Current Experiments

This warm-water problem was only noticed a few months ago, and very little research has gone into it so far. Although considerable past research has been done on the sedimentation tank itself and the plate settlers, nothing has been done on the convective flows encountered here. The convective flows in the plate settler are of particular interest because it does not appear that particles settle as easily in a convective flow as they would in a non-convective case.

When others first addressed this issue, the first steps they took were to try to mimic the convection in a laboratory sedimentation tank. A warm burst of water was introduced into the tank and dyed red. Pictures were taken as the warm water flowed up and out through the tube settlers used in the laboratory apparatus. The results are shown in Figures 3 and 4.

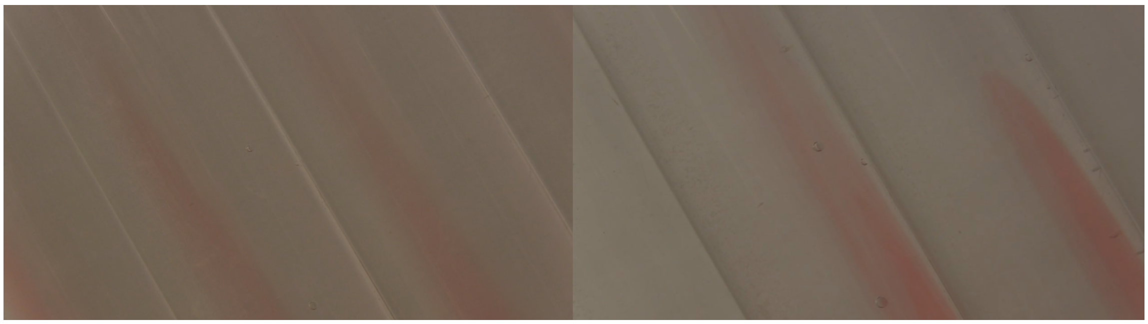

Figure 3: The red plume in the right picture is warmer than the rest of the water. The red plume in the left picture is the room-temperature.

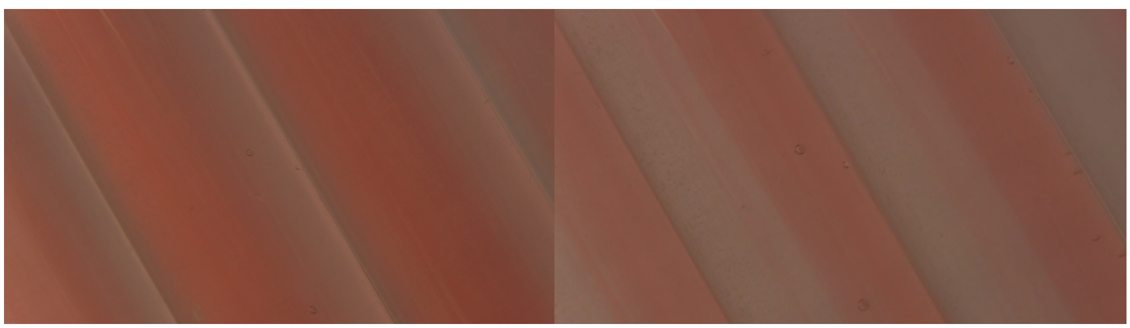

Figure 4: The same experiment, but a few minutes later

Notice how the warm plume traveled along the top side of the slanted tube, while the room-temperature plume did not. From a qualitative perspective, this is exactly what is expected. The warmer water is less dense than the cooler water, thus it is more buoyant and will tend to float above the cooler water. In our case of the plate settlers, we should expect the warm water to be concentrated along the top plate (and flowing upwards due to convection) and the cool water to be concentrated along the lower plate (and flowing downwards due to convection). Since there is a net flow upwards through the plate settlers, we expect that the velocity of the warm-upward flow will be greater than that of the cool-downward flow. Figure 5 shows the general directions of flow in a tube filled with flocs and having a temperature gradient. If observed in person, it is easy to see that the upward velocity flow is quicker than the downward velocity flow.

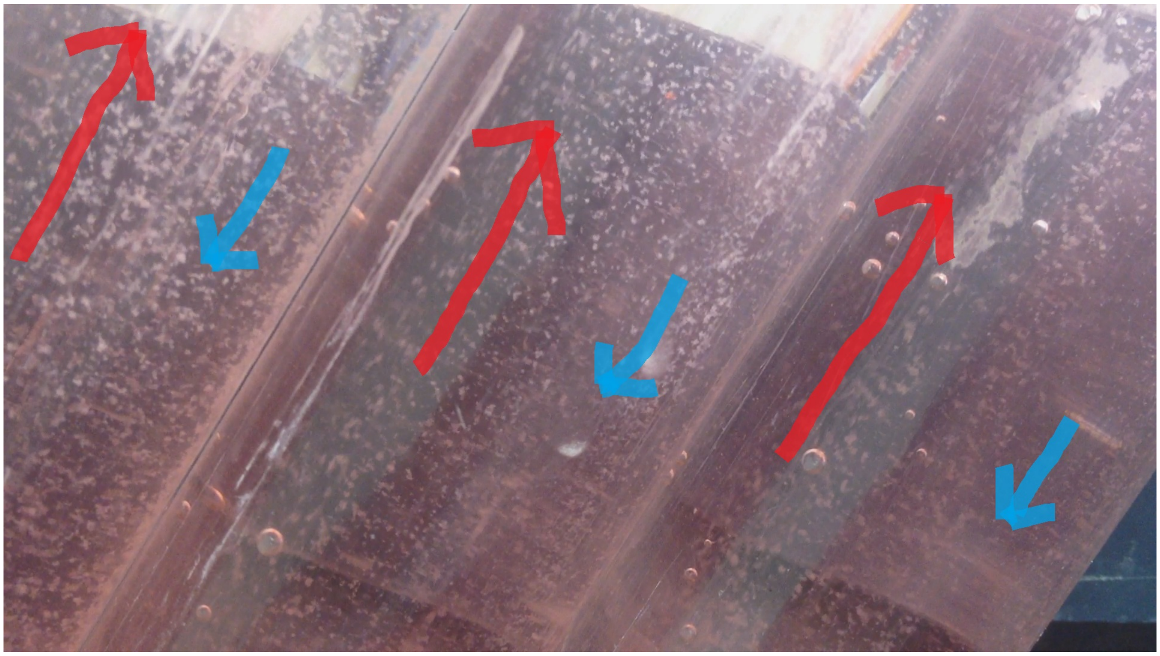

Figure 5: Flocs in a tube with convective flow

More recently, a group has developed a lab setup specifically designed for tests on this subject. They are using a long tube as their settler and are able to precisely control the temperature of the influent water. They have not communicated their findings yet, but they have posted a video of the tube settler running with flocs in it [6].

3 Analytical Model of Flow

The next step in this analysis is to derive an analytical model for the fluid flow through the plate settlers, properly accounting for the increasing temperature of the influent flow. First it is useful to look at the base case in which there is no temperature gradiant. Past research [2] has found that the flow follows the same derivation as Poiseuille Flow in 2d and has a similar result. The flow is given by the equation

$$\vec{u} = u_x(y)\hat{x} = \frac{6V_{\uparrow}}{D^2 \sin\phi}(\frac{D^2}{4} - y^2)\hat{x} \tag{2}$$

where the geometry of the system is shown in Figure 6. The x-axis is along the direction of flow parallel to the plates, the y-axis is perpendicular to the plates with $y = 0$ chosen at the midpoint of between the plates, L is the length of the plates, and D is the separation between the plates. $V_{\uparrow}$ is the average vertical velocity of flow through to plate settlers (typically 1 mm/s) and ϕ is the angle of the plates with respect to horizontal (typically 60°).

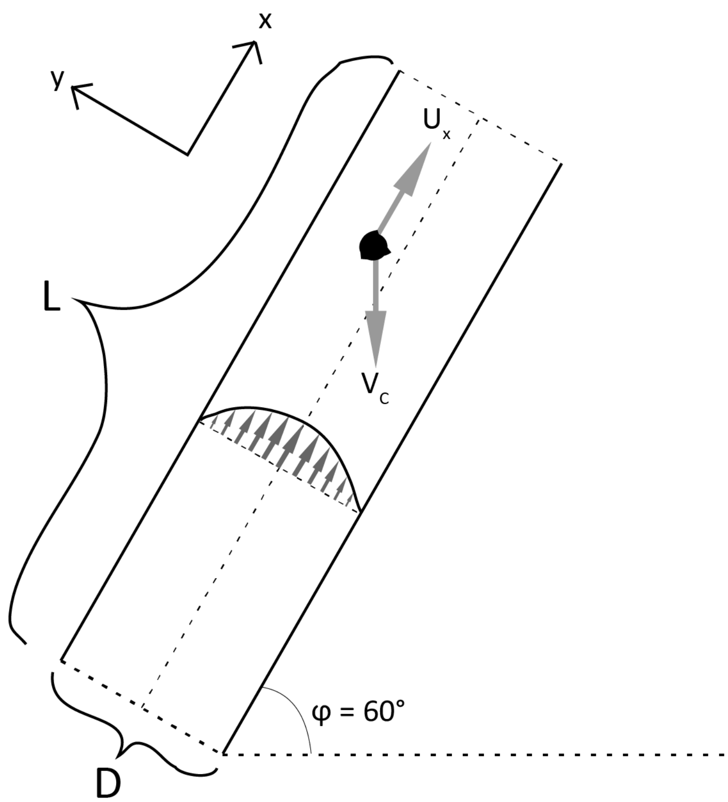

Figure 6: Prediction of flow ignoring convection

Now it is necessary to consider the case without constant temperature such that convection occurs. In the previous section we discussed the qualitative features of this flow, so now we will develop the quantitative model. These calculations only work with a laminar flow. Although we do not have any proof that this flow is laminar, the observations of the earlier experiments imply that it is predominantly laminar. There may be some turbulent regions, however, and those will be discussed later. Since the equations for fluid flow, heat conduction, and convection are all very complex and non-linear, we must make several approximations to obtain a closed-form solution.

The first approximation is the Boussinesq approximation. The Boussinesq approximation takes the temperature of the fluid to be constant for all intents and purposes except when considering its effect on

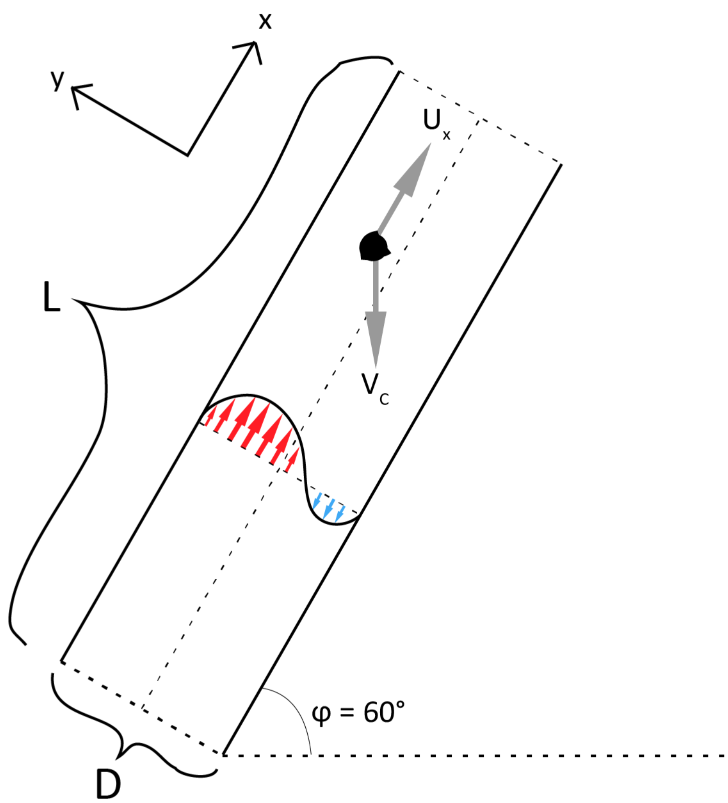

Figure 7: Prediction of flow including convective effects

buoyancy [3, pg 163]. This is necessary because we are now dealing with variations in temperature that may cause changes in the fluid properties. For example: density, viscosity, and thermal conductivity may all be functions of temperature. The Boussinesq approximation states that the only significant effect of temperature is its effect on density. Essentially, we are approximating that a temperature change modifies a fluids density and nothing else, and that the only effect of a change in density is a change in buoyancy. Keep in mind, the changes in ρ are small enough that we are still requiring the fluid be incompressible.

Next, we are dealing with small temperature gradients for which water's density dependance on temperature is approximately linear. Thus, we will take $\Delta\rho$, the change in density, to be linear with ΔT, the change in temperature. More precisely, $\Delta\rho = -\alpha\rho_0\Delta T$, where α is the coefficient of thermal expansion of water and ΔT is the deviation from the average temperature of the fluid.

Before we consider the last approximation, once again consider the geometry of the plates shown in

Figure 6 and Figure 7. In Figure 7, there is a qualitative sketch of the predicted flow pattern including convection, in accordance with the discussions in the previous section. To re-iterate, we expect that when convection is present, there will be a warm column of water traveling up near the top plate, and a cold column of water traveling down near the bottom plate, and the upward column will have more total flow than the downward column such that there is net upward flow through the plate settlers.

The last approximation is that the flow and parameters are independent of x. We have two reasons for believing this to be true. First, this could be visually observed in the experiments in the previous section. It was clear from the videos that the flow patterns at the influent ends of the tubes were very similar to the flow patterns at the effluent ends. Thus, it is reasonable to believe that the fluid velocities and all other parameters mainly depend on y, not x. Second, the ratio of advection to conduction [3, pg 174] (which is a function of the Prandtl number and Grashof number) is high for this scenario, so we would expect that the effects of conduction within the flow are not significant. Combined with the requirements for continuity and incompressibility, this implies that there is nothing that could cause a significant change in flow as the water flows through the plates.

Now that we have outlined the approximations, we have a set of equations that we can use to solve for the flow, from [3, pg 171]

$$\nabla \cdot \vec{u} = 0 \tag{3}$$

$$\vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho}\nabla P + \nu \nabla^2 \vec{u} - \vec{g}\alpha\Delta T \tag{4}$$

$$\vec{u} \cdot \nabla T = \kappa \nabla^2 T \tag{5}$$

Where ν is the kinematic viscosity of water, κ is the thermal diffusivity of water, and $\vec{g}$ is the acceleration due to gravity.

From the last approximation, we know that $\vec{u}$ is only a function of y. Therefore, $\vec{u} = u_x(y)\hat{x} + u_y(y)\hat{y}$. Plug this into (3) and we get

$$\frac{\partial u_y(y)}{\partial y} = 0 \tag{6}$$

The boundary conditions at the plates require $u_y = 0$ there, thus $u_y = 0$ everywhere and $\vec{u} = u_x(y)\hat{x}$

We can now plug this $\vec{u}$ into (4) and (5) to get

$$0 = -\frac{1}{\rho}(\frac{\partial P}{\partial x}\hat{x} + \frac{\partial P}{\partial y}\hat{y}) + \nu\frac{\partial^2 u_x}{\partial y^2}\hat{x} - \vec{g}\alpha\Delta T \tag{7}$$

$$u_x\frac{\partial T}{\partial x} = \kappa(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) \tag{8}$$

Due to the inclination of the axes, $\vec{g} = g\cos(\phi)\hat{y} - g\sin(\phi)\hat{x}$

We can substitute this into (7) and separate for the $\hat{x}$ and $\hat{y}$ components to get two equations

$$0 = \frac{1}{\rho}\frac{\partial P}{\partial y} + g\cos(\phi)\alpha\Delta T \tag{9}$$

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u_x}{\partial y^2} + g\sin(\phi)\alpha\Delta T \tag{10}$$

From our third approximation, we are assuming ΔT does not depend on x, thus $0 = \frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial x^2}$

And consequently, from (8), $\frac{\partial^2 T}{\partial y^2} = 0$, thus ΔT has the form $\Delta T = Ay + B$

Thus there is a linear relationship between ΔT and y. Since the warmer fluid is above the cooler fluid, A must be positive and ΔT will have a maximum at the top plate and a minimum at the bottom plate. Since ΔT is defined as the deviation from the average temperature of the fluid, and since the temperature is approximated as being independent of x, ΔT must equal 0 at the midpoint between the plates. Since we have the freedom to choose where $y = 0$, we can choose $y = 0$ at the midpoint. This forces $B = 0$ and the expression for ΔT becomes $\Delta T = Ay$

The value of A is not arbitrary. A is defined as $(T_{max} - T_{min})/D$ where T_{max} and T_{min} are the maximum and minimum temperatures within the fluid, respectively.

Using this new definition for ΔT, (10) becomes

$$\nu\frac{\partial^2 u_x}{\partial y^2} = \frac{1}{\rho}\frac{\partial P}{\partial x} - g\sin(\phi)\alpha Ay \tag{11}$$

This can be integrated twice to solve for u_x as a function of y

$$u_x(y) = -\frac{g\sin(\phi)\alpha A}{6\nu}y^3 + \frac{1}{2\nu\rho}\frac{\partial P}{\partial x}y^2 + C_1 y + C_0 \tag{12}$$

Where C_1 and C_0 are the integration constants. It is important to note that we actually have three arbitrary constants in this expression, not just two. In addition to the integration constants, $\frac{\partial P}{\partial x}$ is also not yet known. It is convenient that we have three unknowns, because we also have three boundary conditions. The first two boundary conditions are from the no-slip condition, and they require that $u_x = 0$ at the plates. More specifically

$$0 = u_x(-\frac{D}{2}) = u_x(\frac{D}{2}) \tag{13}$$

$$0 = \frac{g\sin(\phi)\alpha A}{48\nu}D^3 + \frac{1}{8\nu\rho}\frac{\partial P}{\partial x}D^2 - \frac{C_1}{2}D + C_0 \tag{14}$$

$$0 = -\frac{g\sin(\phi)\alpha A}{48\nu}D^3 + \frac{1}{8\nu\rho}\frac{\partial P}{\partial x}D^2 + \frac{C_1}{2}D + C_0 \tag{15}$$

The last boundary condition requires that there be a net upward flow through the plate settlers. There is a net positive flow into the tank below the plate settlers, thus continuity and conservation require there be a net flux up and out through the plate settlers. The average vertical velocity of the flow between the plates must equal some defined parameter $V_\uparrow$, which has typically been 1 mm/s in past AguaClara applications.

The average velocity of the fluid in the vertical direction is given by

$$V_\uparrow = \frac{\sin(\phi)}{D}\int_{-D/2}^{D/2} u_x dy = \frac{\sin(\phi)}{D}\int_{-D/2}^{D/2} \left(-\frac{g\sin(\phi)\alpha A}{6\nu}y^3 + \frac{1}{2\nu\rho}\frac{\partial P}{\partial x}y^2 + C_1 y + C_0\right)dy \tag{16}$$

$$= \sin(\phi)\left(\frac{1}{24\nu\rho}\frac{\partial P}{\partial x}D^2 + C_0\right) = V_\uparrow \tag{17}$$

We have three equations (14), (15), and (17) with three unknowns, so solve for C_1, C_0, and $\frac{\partial P}{\partial x}$

$$\frac{\partial P}{\partial x} = -\frac{12\nu\rho V_\uparrow}{\sin(\phi)D^2} \tag{18}$$

$$C_1 = \frac{g\sin(\phi)\alpha A D^2}{24\nu} \tag{19}$$

$$C_0 = \frac{3V_\uparrow}{2\sin(\phi)} \tag{20}$$

Which can be plugged into 12 to yield

$$u_x(y) = -\frac{g\sin(\phi)\alpha A}{6\nu}y^3 + \frac{1}{2\nu\rho}\left(-\frac{12\nu\rho V_\uparrow}{\sin(\phi)D^2}\right)y^2 + \left(\frac{g\sin(\phi)\alpha A D^2}{24\nu}\right)y + \left(\frac{3V_\uparrow}{2\sin(\phi)}\right) \tag{21}$$

$$u_x(y) = \frac{g\sin(\phi)\alpha A}{6\nu}\left(\frac{D^2}{4}y - y^3\right) + \frac{6V_\uparrow}{\sin(\phi)}\left(\frac{1}{4} - \frac{y^2}{D^2}\right) \tag{22}$$

(22) is the solution and completely specifies the the 2d flow between the plates. The only tricky part of applying this equation is determining the value of A, which is defined by $(T_{max} - T_{min})/D$. This value depends on a lot of factors including the geometry of the tank and the heat flow around the tank and is not trivial to determine.

For common cases, the flow has a similar shape to the flow in Figure 7. It consists of a sum of an odd function and an even function. The even terms are due to the net flow up through the plate settler, and the odd terms are due to convection.

Let's apply the solution to several extreme cases and see what we get. First, consider the case when the temperature is constant everywhere. Then, $A = 0$ and the solution simplifies to

$$u_x(y) = \frac{6V_\uparrow}{\sin(\phi)}\left(\frac{1}{4} - \frac{y^2}{D^2}\right) \tag{23}$$

Which is exactly the same as (2), as it should be! This is the parabolic, isothermal flow pictured in Figure 6.

Next, let's consider the case where there is no net flow up through the plates, only convection. In this case, $A \neq 0$ and $V_{\uparrow} = 0$. The solution simplifies to

$$u_x(y) = \frac{g\sin(\phi)\alpha A}{6\nu}(\frac{D^2}{4}y - y^3) \tag{24}$$

The flow for this is pictured in Figure 8. A key takeaway here is that the flow is symmetric about the midline between the plates. This makes sense, because since there is no net flow, the upwards flow must exactly cancel the downwards flow.

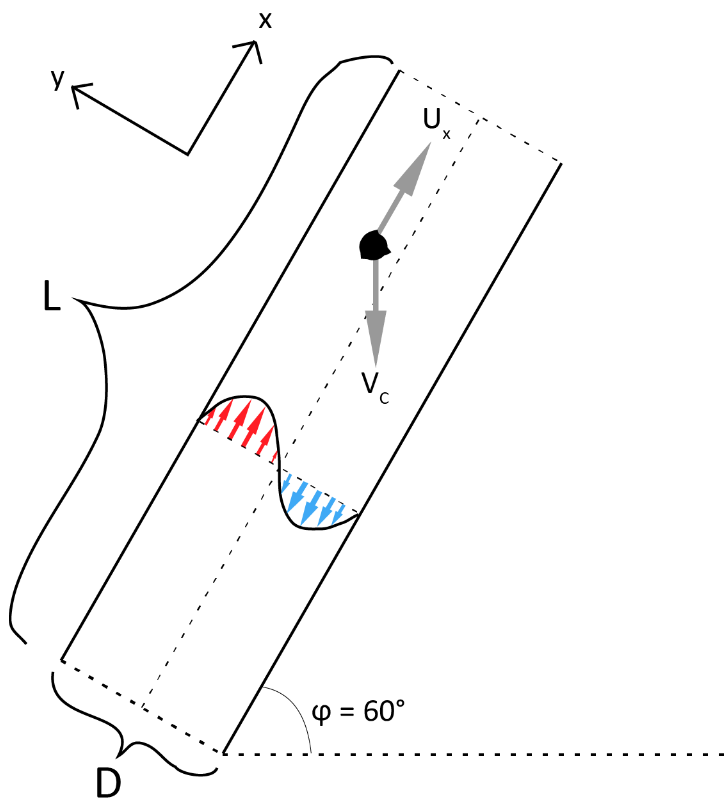

Figure 8: Prediction of flow with only convective effects

4 Turbulence and Shear Instabilities

This model does not provide a complete picture. There is most likely some form of turbulence or chaos in the boundary between the upward and downward flows. If the flow were completely laminar and followed the expression above, there would be no loss of performance with the addition of convection.

Convection would increase the velocity of the upward flowing stream, but it would also decrease its width, which can be seen in the solution above. Thus, a floc caught in the stream would travel faster, but it would also have a shorter distance to fall before getting caught in the cooler, downward stream. When the flocs reach the boundary between the two streams, if the boundary were laminar, they would smoothly transition into the downward stream and be carried back into the sedimentation tank.

However, empirically, that doesn't happen. Observations of the lab experiment tell us that flocs are actually less likely to settle when there's convection and two opposing flows. This suggests some form of turbulence or chaos that carries flocs back up into the upward flow when they reach the boundary.

The literature tells us that this is not a far-fetched idea and it is reasonable to expect some sort of turbulence or chaos at the boundary. The geometry we are using creates a situation very similar to Rayleigh convection in a vertical slot. The fluid near one of the plates is at a higher temperature than the other, and this drives the convection of fluid closer to the midpoint between the two plates. Rayleigh convection between two vertical walls is discussed in [3, pg 37], and he concludes that turbulence can occur in the middle of the slot for conditions with high enough Rayleigh numbers.

Likewise, Kelvin-Helmholtz instability will cause an oscillation and potentially growing perturbation at the boundary. As stated in [], any case with opposing flows is unstable.

5 Larger Flow Patterns and 3d Considerations

All the previous discussion has only considered the flow between a pair of plates. This is appropriate for discussion with the lab experiments, because the lab experiments used a small, similar geometry. it is good place to start and get a general idea of the problem and possible flow patterns, but it doesn't reflect the entire geometry of the problem. When the entire geometry of the sedimentation tank is considered, other flow patterns are possible that wouldn't be possible with just a single pair of plates.

For example, in the previous discussion, we considered the possibility of a warm column of water and a cool column of water between the same pair of plates, with the warm column flowing upwards and the cool column flowing downwards. However, there is nothing requiring that the two columns must be between the same pair of plates. Consider the flow in Figure 9. In this configuration, warm columns flow up between one pair of plates and flow down through an adjacent pair. As before, the velocity up through the warmer columns must be greater than the velocity down through the cooler columns, such that there is a net upward flow.

It is useful to figure out how such a flow pattern would affect the performance of the plate settlers,

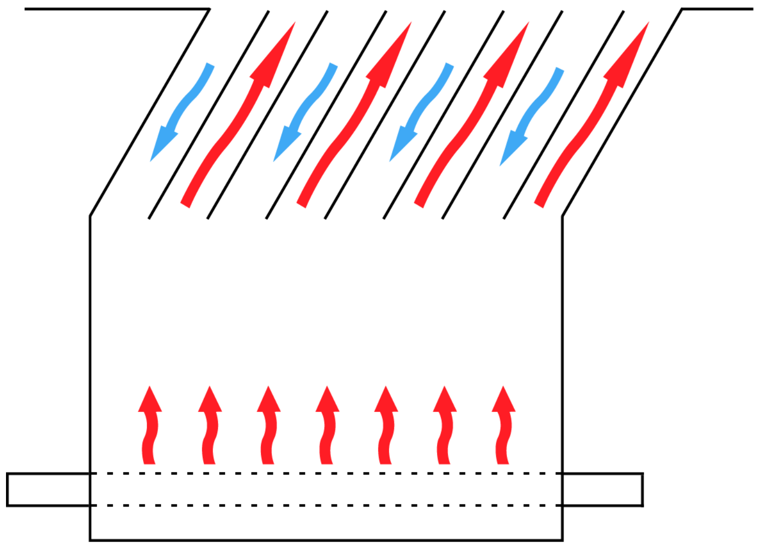

Figure 9: Another possible flow pattern in the sedimentation tank

should it occur. One notable thing about this configuration is that there is no shear flow, despite the convection. The upward column of warm water is isolated from the cooler column as they pass through the array of plates. Thus, it is reasonable to expect that they both follow laminar Poiseuille flow as they travel through the plates, just like the flow indicated in Figure 6. For a case like this, it is very easy to solve for the capture velocity of flocs, and it has already been done in (1). Referring back to the earlier discussion, a higher vertical velocity in the column will lead to a high capture velocity, preventing the column from catching smaller flocs.

Thus, this offers another explanation for why the plate settlers are not performing well when warm water is introduced to the sedimentation tank. If the flow follows the pattern in Figure 9, there will be several channels through which very high velocity warm water passes through the plate settlers. This high velocity does not give the flocs a chance to settle, causing them to get whisked past the plate settlers and out of the sedimentation tank. Unfortunately, the math gets fairly complicated for a case like this, and it isn't easy to solve for the precise velocity.

The above was just one example of a possible flow pattern in the sedimentation tank. There are likely many others that have not been mentioned. In fact, even more flows are possible when you consider 3d flows. One such possibility is pictured in Figure 10. Water is introduced to the sedimentation tank through a pipe in the bottom-center of the tank. If this water is warm and buoyant, it will rise straight up and through the plate settlers, resulting in the flow pattern pictured. Just like the last example,

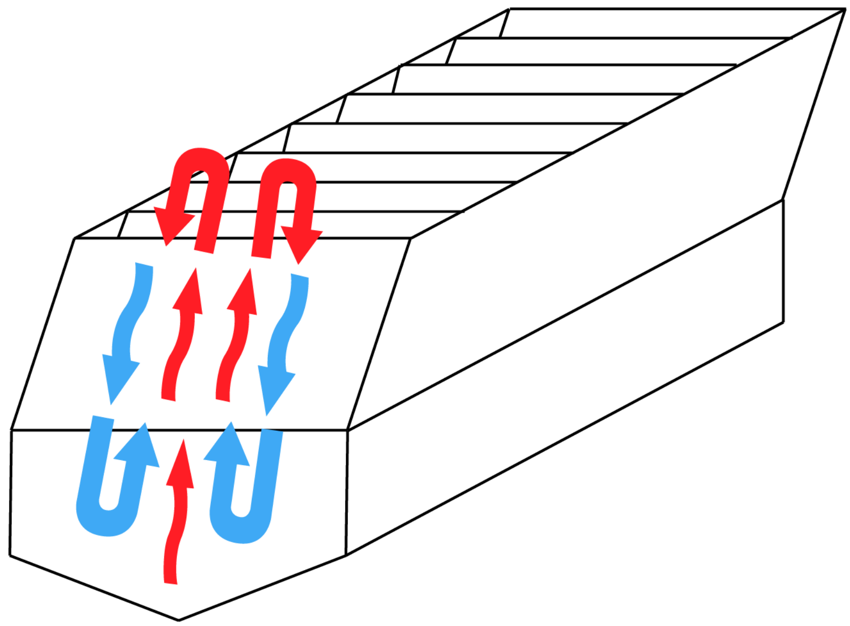

Figure 10: Another possible flow pattern when considering 3d

this may result in a warm, high velocity channel through which flocs are whisked up and out of the sedimentation tank.

6 Conclusion

The water flow in the AguaClara settling plates was investigated in the case of convection. The convection was brought about by warm, sun-heated water entering the sedimentation tank and triggering a significant drop in performance.

An analytical solution was found for 2d convective flow between two angled walls in the presence of a net upward flow. This solution might help explain some of the laboratory results, as the geometry is very similar to that used in the lab.

The analytical solution assumed laminar flow and predicted that flocs would settle just as easily under convective flow as they would under non-convective flow. Previous observations of the laboratory experiment indicated that this is not the case, and that the flow patterns produced by convection are not conducive to sedimentation. In particular, visual observation seems to suggest some sort of turbulence at the boundary where the warm and cool flows met. The literature agrees that this is possible. For everywhere other than the boundary, however, the analytical solution should be accurate.

Due to the more complicated geometry of the actual AguaClara sedimentation tank, the flow may not follow the same patterns as it does in the laboratory tube-settlers. There is a potential for larger scale convection currents that could not happen on a lab bench. Such large-scale flow patterns may also be the source of the performance hit. In the event that the laboratory tests are not able to solve the performance issues, it may be helpful to investigate whether some large scale flow is occurring in the full-sized sedimentation tank.

7 References

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