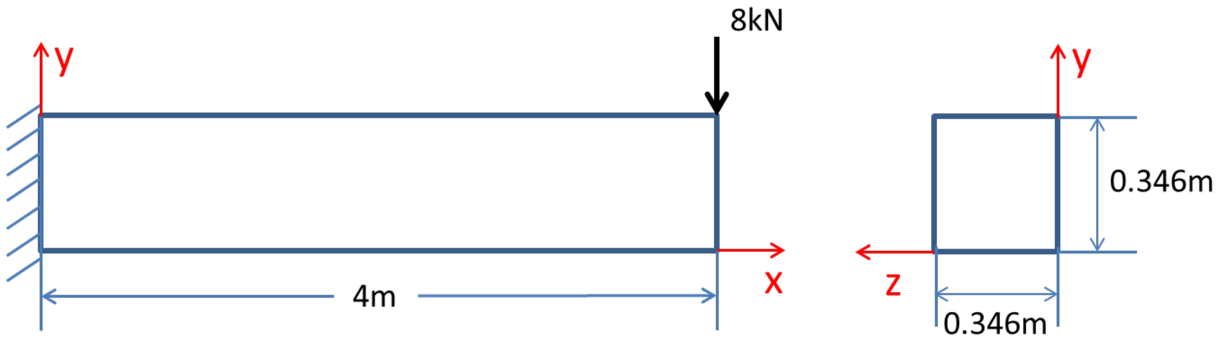
Modeling a Cantilever Beam using Beam Theory, 2D Elasticity and 3D Elasticity

The learning goals of this exercise are to:

- Study the relative efficiency and accuracy of three different approaches to modeling a cantilever beam in bending
- Study the effect of the stress singularity at the fixed end of the beam

Consider the cantilever beam shown below which is modeled using Euler-Bernoulli beam theory in the tutorial at *https://confluence.cornell.edu/x/9L9-Bw*. The beam is clamped on the left end and has a force of 8kN acting downward at the right end. This 8 kN force acts over the entire cross-section at the right end. The beam has a length of 4 meters, width of 0.346 meters and height of 0.346 meters (the cross-section is a square). Additionally, the beam is composed of a material which has a Young's Modulus of 28 GPa and a Poisson's ratio of 0.3.



In this exercise, you need to compare the results from modeling this beam using three different approaches:

1. Euler-Bernoulli beam theory or "beam theory" for short (denoted as "1D" below)
2. 2D elasticity with plane stress
3. 3D elasticity

The first approach is covered in the beam tutorial mentioned above. You'll have to develop ANSYS models for the other two approaches.

Use the following meshes: 2x2x2, 4x4x4, 8x8x8, 16x16x8, 64x64x8. For instance, in the case of the mesh denoted as 4x4x4, you'll have:

- 4 divisions for the line in the beam theory model (4 line elements)
- 4 divisions in each direction for the area in the 2D model (4x4=16 area elements)
- 4 divisions in each direction for the volume in the 3D model (4x4x4 = 64 volume elements)

Note that for the 3D model, the number of divisions in the z-direction is not increased beyond 8 to keep the mesh size reasonable. To get a regular hexahedral mesh for the 3D model, you will need to use the sweep method where one face is meshed and then this surface mesh is extruded to get the volume mesh. To apply the sweep method, select *Mesh > Mesh Control > (Select entire body) > Apply > Method > Sweep*.

Part 1: Mathematical Models

For each modeling approach, state the:

a. The mathematical model
b. The fundamental physical principle on which the model is based
c. Key assumptions in the model

In part (a), state the governing equations, auxiliary equations and boundary conditions for the 2D and 3D elasticity models. For beam theory, state the integral equation that forms the basis for deriving the discrete equations.

Part 2: Finite-Element Solution

For each model and mesh, determine:

- Displacement in the y direction at the right end of the beam in the middle of the cross-section (x = 4 m, y = 0.346/2 m, z = 0.346/2 m). Let's denote this displacement as v_1.
- Stress σ_{xx} distribution at the left end ($x = 0$) and in the middle ($x = 2\ m$). Note the maximum value of σ_{xx} over the cross-section at each location.

Summarize the total number of degrees of freedom (DOF) and the displacement v_1 in the form of the following table:

Mesh	DOF (1D)	v_1 (1D)	DOF (2D)	v_1 (2D)	DOF (3D)	v_1 (3D)
2x2x2						
4x4x4						
8x8x8						
16x16x8						
64x64x8						

For 1D elements, while calculating the total degrees of freedom, you can ignore the z-displacement DOF and the rotational DOF about x and y axes that ANSYS puts in. These are identically zero for this case.

Summarize the results for the maximum value of σ_{xx} at x=2m and x=0m in the form of the following table:

Mesh	Max. σ_{xx} at x=2m (1D)	Max. σ_{xx} at x=2m (2D)	Max. σ_{xx} at x=2m (3D)	Max. σ_{xx} at x=0m (1D)	Max. σ_{xx} at x=0m (2D)	Max. σ_{xx} at x=0m (3D)
2x2x2						
4x4x4						
8x8x8						
16x16x8						

64x64x8						

Report the displacements and stresses to *four* significant digits. For units, use meters for displacement and MPa for stresses.

Using the filled-in tables, respond to the following questions:

- How many degrees of freedom do you need to use with 1D, 2D and 3D elements to get the above displacement and stress values at x = 2m to an accuracy of *three* significant digits?
- How well do the above displacement and stress values agree between the 1D, 2D and 3D models?
- Which converges faster on refining the mesh: displacement or stress? Why?
- What is the effect of the stress singularity at the fixed end (x=0m) on the above displacement and stress values as the mesh is refined? What is the difference in results at the fixed end between the 1D, 2D and 3D models?
- What are your conclusions about the relative efficiency and accuracy of the 3 different modeling approaches?

For tips on how to plot the stress distribution in any beam cross-section and how to determine the displacements at any location, see the video at *https://youtu.be/Nq1qLWIqICU*