Pre-Analysis

1 σ_x

First, let's begin by finding the average stress, the nominal area stress, and the maximum stress with a concentration factor.

$$\sigma_o = \frac{F}{A} = \frac{1000000}{.2 \times 5}\frac{\text{lb}}{\text{in}^2} = 1 \times 10^6 \text{ psi}$$

$$\sigma_{nominal} = \frac{F}{A} = \frac{1000000}{.2 \times (5 - .5)}\frac{\text{lb}}{\text{in}^2} = 1.111 \times 10^6 \text{ psi}$$

For a finite plate with a hole, there is no analytical solution. However, an analytical solution does exists for an infinite plate with a hole. The concentration factor for an infinite plate with a hole is K = 3. The maximum stress for an infinite plate with a hole is

$$\sigma_{max} = K \times \sigma_o$$

$$\sigma_{max} = (3.0)(1.0 \times 10^6 \text{psi}) = 3.0 \times 10^6 \text{psi}$$

Although there is no analytical solution for a finite plate with a hole, there is empirical data available to find a concentration factor. Using a Concentration Factor Chart (3250 Students: See Figure 4.22 on page 158 in *Deformable Bodies and Their Material Behavior*), we find that $\frac{d}{w} = .1$ and thus $K \approx 2.73$ Now we can find the maximum stress using the nominal stress and the concentration factor

$$\sigma_{max} = K \times \sigma_{nominal} = (2.73)(1.111 \times 10^6 \text{ psi}) = 3.033 \times 10^6 \text{ psi}$$

2 σ_r

Now, let's look at the radial stress varies in the plate:

$$\sigma_r(r, \theta) = \frac{1}{2}\sigma_o[(1 - \frac{a^2}{r^2}) + (1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2})cos(2\theta)]$$

at r=a

$$\sigma_r = 0$$

assuming r $\gg$ a

$$\sigma_r(r, \theta) = \sigma_r(\theta) = \frac{1}{2}\sigma_o[1 + cos(2\theta)]$$

Now we will examine the stress far from the hole at $\theta = 0$ (the x-axis) and $\frac{\pi}{2}$ (the y-axis)

$$\sigma_r(0) = \frac{1}{2}\sigma_o[1 + cos(2(0))] = \sigma_o$$

$$\sigma_r(\frac{\pi}{2}) = \frac{1}{2}\sigma_o[1 + cos(2(\frac{\pi}{2}))] = 0$$

3 σ_θ

Now we will examine how σ_θ varies in the plate. We will approach this very similarly to how we approached the examination of σ_r:

$$\sigma_\theta(r, \theta) = \frac{1}{2}\sigma_o[(1 + \frac{a^2}{r^2}) - (1 + 3\frac{a^4}{r^4})cos(2\theta)]$$

at r = a

$$\sigma_\theta = \frac{1}{2}\sigma_o(2 - 4cos(2\theta))$$

assuming r $\gg$ a

$$\sigma_\theta(r, \theta) = \sigma_\theta(\theta) = \frac{1}{2}\sigma_o[1 - cos(2\theta)]$$

Now we will examine the stress far from the hole at $\theta = 0$ and $\frac{\pi}{2}$

$$\sigma_\theta(0) = \frac{1}{2}\sigma_o[1 - cos(2(0))] = 0$$

$$\sigma_\theta(\frac{\pi}{2}) = \frac{1}{2}\sigma_o[1 - cos(2(\frac{\pi}{2}))] = \sigma_o$$

4 $\tau_{r\theta}$

Finally, we will examine how the shear stress in the $r\theta$ direction varies in the plate. The equation for the shear stress in the plate is:

$$\tau_{r\theta} = -\frac{1}{2}\sigma_o(1 - 3\frac{a^4}{r^4} + 2\frac{a^2}{r^2})sin(2\theta)$$

at r = a

$$\tau_{r\theta} = 0$$

assuming r $\gg a$

$$\tau_{r\theta}(r,\theta) = \tau_{r\theta}(\theta) = -\frac{1}{2}\sigma_o sin(2\theta)$$

Now we will examine the values of $\tau_{r\theta}$ when r $\gg$ a and at $\theta = 0$ and $\theta = \frac{\pi}{2}$

$$\tau_{r\theta}(0) = -\frac{1}{2}\sigma_o sin(2(0)) = 0$$

and

$$\tau_{r\theta}(\frac{\pi}{2}) = -\frac{1}{2}\sigma_o sin(2(\frac{\pi}{2})) = 0$$

We will reexamine all of these calculations so we may estimate the validity of the ANSYS simulation later in this tutorial.