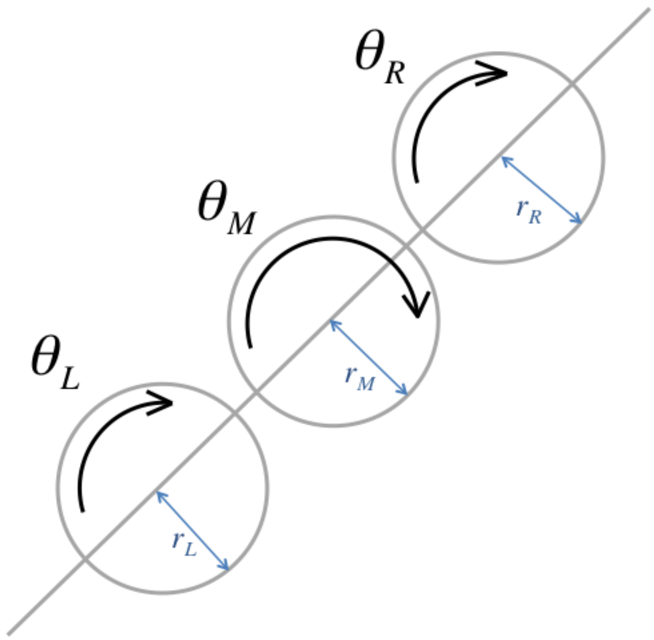
Subscripts L, R, M correspond the left half-shaft, right half-shaft, and middle gear, respectively.

Figure 1: Each grey circle represents a different gear on the differential.



Three observations:

1. When we hold the right axle fixed and turn the left axle one revolution, the middle sprocket makes half a revolution in the same angular direction.
2. Holding the middle sprocket in place and turning the left axle one revolution causes the right axle to make one revolution in the opposite direction.
3. The above observations are the same if the left and right axle are switched.

The results can be summarized as such:

$$\theta_{\mathrm{M}} = \frac{1}{2}\theta_{\mathrm{L}} + \frac{1}{2}\theta_{\mathrm{R}} \tag{1}$$

where θ represents angular position, ω represents angular velocity, α represents angular acceleration, and τ represents torque. r is the radius of the gear, and v is the linear velocity at the edge of a gear.

$$\begin{aligned}
\omega_{\mathrm{M}} &= \frac{1}{2}\left(\omega_{\mathrm{L}} + \omega_{\mathrm{R}}\right) &(2)\\
\omega_{\mathrm{M}} r_{\mathrm{M}} &= \frac{1}{2}(\omega_{\mathrm{L}} r_{\mathrm{M}} + \omega_{\mathrm{R}} r_{\mathrm{M}}) &(3)\\
v_{\mathrm{M}} &= \frac{1}{2}\left(\omega_{\mathrm{L}} r_{\mathrm{M}} \frac{r_{\mathrm{L}}}{r_{\mathrm{L}}} + \omega_{\mathrm{R}} r_{\mathrm{M}} \frac{r_{\mathrm{R}}}{r_{\mathrm{R}}}\right) &(4)\\
v_{\mathrm{M}} &= \frac{1}{2}\left(v_{\mathrm{L}} \frac{r_{\mathrm{M}}}{r_{\mathrm{L}}} + v_{\mathrm{R}} \frac{r_{\mathrm{M}}}{r_{\mathrm{R}}}\right) &(5)
\end{aligned}$$

We assume that these ideal[1] gears are frictionless and massless. Therefore, we can use conservation of energy to say that input power equals output power. Say P represents power as a function of time.

$$\begin{aligned}
&P_{\mathrm{M}} = P_{\mathrm{L}} + P_{\mathrm{R}} &(6)\\
&\tau_{\mathrm{M}} \cdot \omega_{\mathrm{M}} = \tau_{\mathrm{L}} \cdot \omega_{\mathrm{L}} + \tau_{\mathrm{R}} \cdot \omega_{\mathrm{R}} &(7)\\
&\tau_{\mathrm{M}} \cdot \left[\frac{1}{2}\left(\omega_{\mathrm{L}} + \omega_{\mathrm{R}}\right)\right] = \tau_{\mathrm{L}} \cdot \omega_{\mathrm{L}} + \tau_{\mathrm{R}} \cdot \omega_{\mathrm{R}} &(8)
\end{aligned}$$

If we consider ω_{L} and ω_{R} separately, we find that

$$\begin{aligned}
\frac{1}{2}\tau_{\mathrm{M}}\omega_{\mathrm{L}} = \tau_{\mathrm{L}}\omega_{\mathrm{L}}, &\quad \frac{1}{2}\tau_{\mathrm{M}}\omega_{\mathrm{R}} = \tau_{\mathrm{R}}\omega_{\mathrm{R}} &(9)\\
\frac{1}{2}\tau_{\mathrm{M}} = \tau_{\mathrm{L}}, &\quad \frac{1}{2}\tau_{\mathrm{M}} = \tau_{\mathrm{R}} &(10)
\end{aligned}$$

Therefore, we can conclude that the relationships between torques is the same as the relationships between angular position: *the torque on the middle gear is evenly divided between the torque on the left gear and the torque on the right gear.*

$$\begin{aligned}
\tau_{\mathrm{M}} = \tau_{\mathrm{L}} + \tau_{\mathrm{R}} &\qquad (11)\\
\tau_{\mathrm{L}} = \tau_{\mathrm{R}} = \frac{1}{2}\tau_{\mathrm{M}} &\qquad (12)
\end{aligned}$$

[1] `http://xkcd.com/669/`