Nonideal concentration of nonimaging linear Fresnel lenses

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ABSTRACT

Our paper discusses geometrical and optical concentration ratios of the optimum nonimaging arched linear Fresnel lens which we designed earlier. This is a fundamental issue, with practical implications for the design of refractive nonimaging concentrators. The deliberations yield a better understanding of the way the refractive index of the thin lens, and the refractive index of the possible dielectricum between lens and receiver, as well as light incident in the plane of the secondary acceptance half angle ψ, influence the performance of the nonimaging concentrator. Theoretical results are compared with tests of the existing prototypes of the nonimaging lens, used for the concentration of solar radiation. The novel nonimaging lens is put into the context of historic research, and is made comparable to other nonimaging concentrators, notably the Compound Parabolic Concentrator. We propose the use of a linear kaleidoskope-based secondary concentrator to achieve a uniform flux distribution and the reproduction of the spectrum of the incoming light.

Keywords: Acceptance half angles, flux uniformity, Fresnel lens, ideal concentration, nonimaging concentration, refractive index

1. INTRODUCTION

Fresnel lenses can be designed as nonimaging lenses.[1] Their design[2] follows the edge ray principle. Two pairs of acceptance half angles, $\pm\theta$ and $\pm\psi$ are defining the window of acceptance. Rays incident on the entry aperture of the lens at angles smaller than the acceptance half angles are refracted towards the absorber, and leave the optical system through its exit aperture. Typically for nonimaging concentrators, rays incident at angles greater than the acceptance half angles generally miss the receiver.

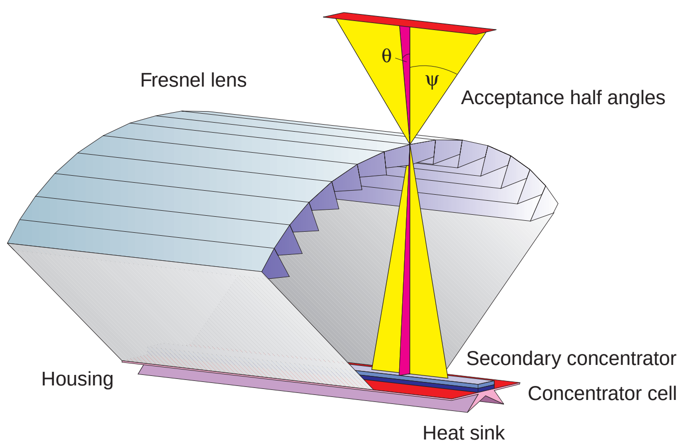

Figure 1. Design study of one-axis tracking terrestrial nonimaging Fresnel lens integrated with photovoltaic concentrator module. Acceptance half angles $\theta = \pm 2°$ in the cross-section (plane of paper), $\psi = \pm 12°$ in the perpendicular plane for a lens with concentration ratio $C = 19.1$. Not to scale.

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Nonimaging Fresnel lens solar concentrators (our prototypes have been built for solar concentration) can be designed with higher tolerance for tracking errors, accounting for effects due to the size of the solar disk, and unproblematic color behavior, than conventional imaging lenses. One-axis azimuthal tracking is practical for a terrestrial lens module of medium optical concentration ratio of approximately 15, translating into acceptance half angle pairs of $\theta = \pm 2°$ and $\psi = \pm 12°$. A schematic of a prototype of the novel nonimaging Fresnel lens is shown in Fig. 1.

2. OPTICAL PROPERTIES OF THE NONIMAGING FRESNEL LENS

The optical properties of nonimaging lenses are characterized by the

- optical efficiency η taking into account reflection losses and geometrical losses;
- geometrical concentration ratio C as ratio of lens surface area to absorber surface area, calculated for nonimaging linear concentrators by the well known relation $C = 1/sin\,\theta$;
- optical concentration ratio, which is understood as the ratio of angular radiation intensity after having passed the lens, and thereafter passing through the second aperture, to the radiation intensity of identical radiation that has not been interfered with: $\eta_C = \eta\, C$, an example is given in Fig. 2;
- flux density distribution ξ on the absorber, in terms of absolute flux at a given absorber location. For solar concentrators, the quality of the reproduction of the solar spectrum on the receiver is significant for the performance of photovoltaic multijunction devices (Fig. 3 and Sect. 3.3).

Note that the concentration ratios on the one hand, and the flux distribution on the other hand are exclusive concepts without direct relation in most cases. Only if the light source is of Lambertian quality, and the lens concentrator can be called ideal, then the absorber sees a Lambertian source, and consequently is illuminated in an ideal way, resulting in an ideal flux distribution, achieved by ideal concentration.

2.1. Optical concentration ratio

Nonimaging concentrators like the CPC or this Fresnel lens can be truncated at half height ($f_t = 1/2f$). The parabolic mirrors or the prisms are cut with minor impact on concentrator performance but significant cost reduction due to material savings. A comparison of truncated and nontruncated lenses has been pictured in Fig. 2 for a stationary lens of acceptance half angles $\theta = \pm 25°$, and $\psi = \pm 35°$, where a wider plateau offers better visualization of the effect.

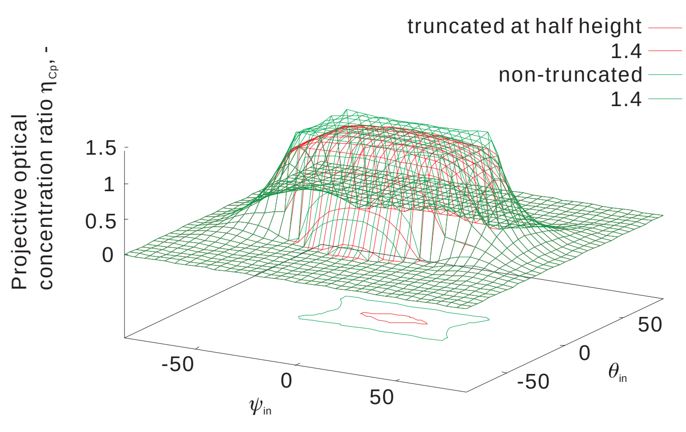

Figure 2. Optical concentration ratio based on the projective concentration ratio of a nonimaging Fresnel lens with acceptance half angles $\theta = \pm 25°$, $\psi = \pm 35°$, incidence angles ψ_{in} and θ_{in}. The same lens is truncated at half height above the absorber for comparison, and shown as inner grid along with the center contour line

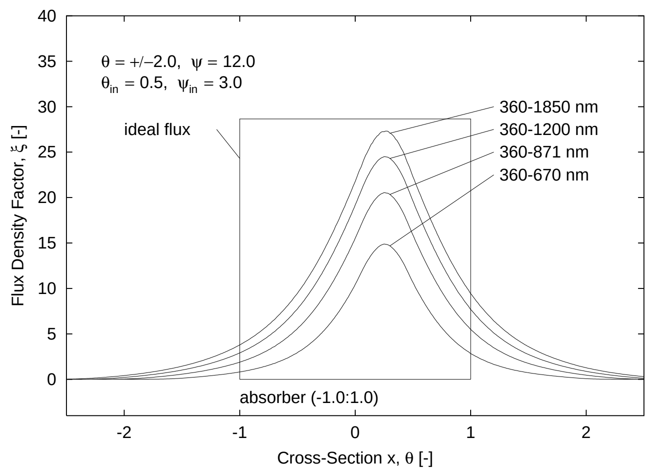

Figure 3. Solar spectral flux reproduction of the nonimaging linear Fresnel lens; design acceptance half angles θ and ψ in degree, incidence in the cross-sectional plane θ_{in}, and in the plane of the paper ψ_{in}. Reasons for the difference between ideal and simulated spectral flux include the finite refractive index of the lens, and truncation of the lens

2.2. Flux density

A typical reproduction of the solar spectrum by an existing nonimaging device, a linear Fresnel lens, is shown in Fig. 3. Clearly, both the local irradiance is changing over the locations over the absorber, and the solar spectrum is not equally reproduced for all locations. The latter is indicated by the relative changes in the vertical distance between the graphs representing cumulative energy for response ranges for light of various wavelengths. Optimum performance of the solar cell can only be expected under ideal flux (Fig. 3). Flux issues, such as the simulation method have been discussed elsewhere.[1,3]

From Figs. 2 and 3, we have to conclude that the nonimaging Fresnel lens is not an ideal concentrator. Although designed according to the edge ray principle as optimum lens, the optical performance of the nonimaging lens has been found to be suboptimal in simulation and experiment. Why?

3. NONIDEAL CONCENTRATION

A concentrator[4] is called 'ideal' when all rays entering the first aperture of the concentrator system within two pairs of acceptance half angles θ and ψ are exiting through the second aperture of the concentrator system over a solid angle of π.

Four issues are to be discussed when determining whether the nonimaging Fresnel lens may be an ideal concentrator, namely

- does the lens act as ideal light source illuminating the receiver, and under what conditions may the lens be considered a Lambertian radiator? The refractive index of the lens, and not only the refractive index of the translucent material between lens and receiver, plays a decisive role in this matter;
- is the idealness of the lens affected by its design as two-dimensional, or three-dimensional concentrator, and what influence does the perpendicular acceptance half angle ψ have?
- to what extent is the performance of the lens practically restricted by total reflection, internal or on the outer surface?
- how does truncation of the lenses outer reaches curtail its performance?

If the light source is ideal ('Lambertian'), being characterized by constant flux over all directions, the absorber must equally be a Lambertian radiator if the concentrator should be called ideal. The sun is an almost ideal radiator, but, unfortunately, the nonimaging Fresnel lens is not.

3.1. Refractive index of infinity

Refraction is characterized by Snell's law, which was essentially announced in 1621 by the Dutch astronomer and mathematician Willebrord Snell (1591–1626). In France, the law is called Descartes' law, since he used the ratio of the sines first. Referring to Fig. 4 for conventions,

$$\frac{\sin\phi}{\sin\phi'} = \frac{n'}{n}\,, \tag{1}$$

with $n' > n$. The refractive index n of materials is defined as the ratio of the speed of light in vacuum $c = 2.997925 \cdot 10^8$ m/s to the speed of light in a medium v. When only one refractive index n_D is given this refers to yellow light at the wavelength $\lambda_\mathrm{D} = 589.2$ nm:

$$n = \frac{c}{v}\,. \tag{2}$$

A ray entering a material of higher refractive index will be refracted towards the normal, a ray leaving a substance of higher refractive index into a material of lower refractive index will be refracted away from the normal.

The nonlinear behavior of Snell's law has been made responsible for asymmetrical input and output angles (ϕ_1 and ϕ_2 in Fig. 4) at the prisms of the lens.[5] Thus, the absorber would not see the lens (and the sun) as Lambertian light source, unless the lens was made out of material with refractive index $n \to \infty$. This explanation assumes that the symmetry of these angles is perfected, once the prism surfaces are almost parallel, allowed by the asymptotic Snell's law with a refractive index approaching infinity. Obviously, ϕ_1 and ϕ_2 are not symmetrical for all prisms of the lens, even though ideal. Still, the ideal nonimaging lens must have $n \to \infty$.

Figure 5 pictures linear nonimaging Fresnel lenses of various refractive indices. The design angles are kept constant at $\theta = \pm 1°$ and $\psi = \pm 6°$. The cases of $n = 1.01$ and $n = 50.0$ are the extreme examples; the latter is almost identical to $n \to \infty$. Of more practical value are the lenses with refractive indices near water ($n = 1.32$), PMMA ($n = 1.49$), flint glass ($n = 1.68$), and diamond ($n = 2.42$).

The shapes of the ideal nonimaging Fresnel lenses ($n \to \infty$) can be concluded based on the following two-dimensional thoughts:

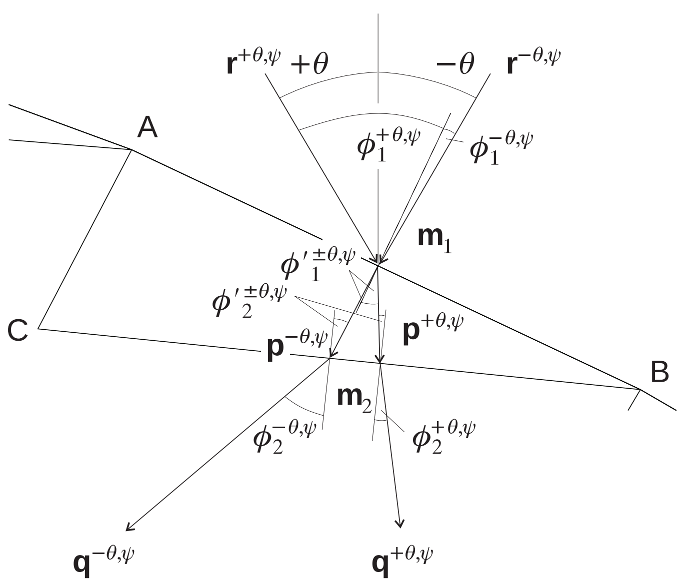

Figure 4. Nomenclature used for the description of reflection and refraction at the prism. The incident rays $\vec{r}_\mathrm{i}$, the rays refracted at the first surface $\vec{p}_\mathrm{i}$, and the rays refracted at the second surface $\vec{q}_\mathrm{i}$ are drawn to scale depending on the extreme acceptance half angles $\pm\theta$, and ψ, the latter being symmetrical. Projection into the cross-sectional plane

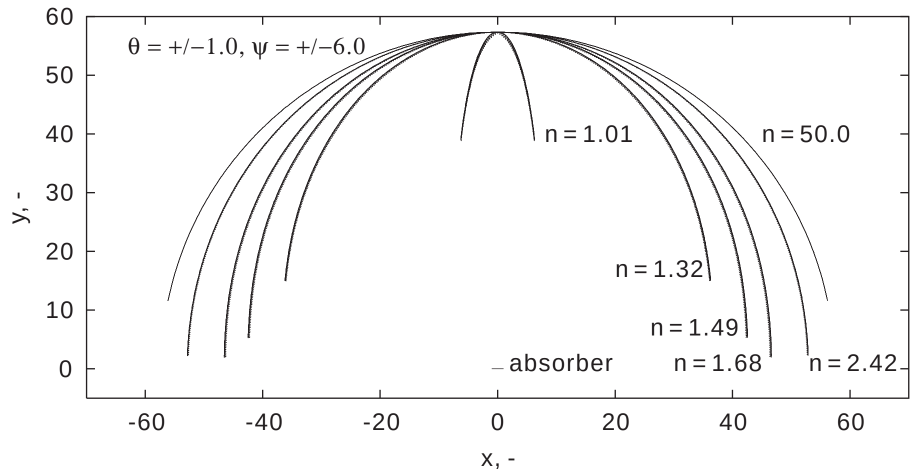

Figure 5. Influence of the refractive index n on the width of the nonimaging Fresnel lens of $\theta = \pm 1°$ and $\psi = \pm 6°$, approaching ideal concentration

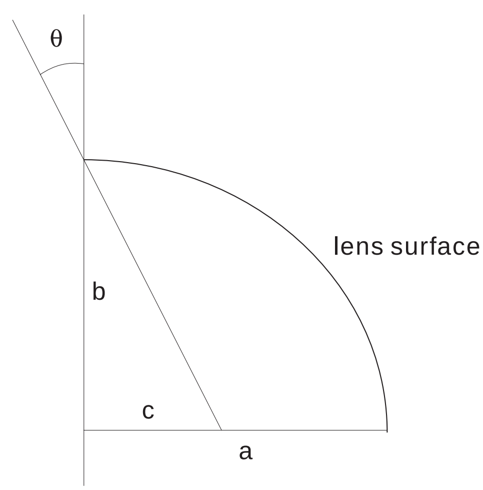

Figure 6. Elliptical shape of the ideal nonimaging Fresnel lens

- The prism should be a prism of minimum deviation, to keep the spread angle of the outgoing beam as small as possible. The spread of the beam directed towards the absorber of a small size determines how far from the absorber the prism can be installed, in order that the extreme rays of the beam meet the edges of the absorber. The front and bottom surfaces of the ideal prism with $n \to \infty$ become almost parallel. For the plane parallel plate, and normal incidence, deviation can be $\delta = 0$, representing the ideal case.
- The refractive power of the prism depends on its refractive index, and on its geometry, i.e. the prism angle β. If the prisms angle can be kept small due to a high index of refraction, the prism's inclination α can be strongly negative: the face of the prism can see the light source, and the bottom of the prism can see the absorber. Again, this leads to the possibility of moving the prism further away from the absorber; the slope of the lens is less steep for larger n at given height over the absorber.

We assume that the ideal lens will have an elliptical shape.[6] In practice, we determine the starting point for the lens design as (see Fig. 6)

$$b = \frac{c}{\tan\theta} , \tag{3}$$

where c is the absorber half width, set to unity. The foci of the ellipse are located at the absorber's edges. The general equation describing the ellipse is

$$c = \sqrt{a^2 - b^2} \,, \tag{4}$$

with a and b being the half axes of the ellipse. From (4) and (3), a can be derived:

$$a = \frac{c}{\sin\theta} \,. \tag{5}$$

This, clearly, is the definition of the geometrical concentration ratio of the ideal two-dimensional nonimaging concentrator,

$$\frac{a}{c} = \frac{1}{\sin\theta} \,. \tag{6}$$

Looking at the lenses in Fig. 5 reveals, that only the lens of $n \to \infty$ is an ideal concentrator. All other lenses are not wide enough. Table 1 lists the geometrical properties of the lenses in Fig. 5. The comparison shows that material choices can make some difference: the expected performances of water-filled lenses, or lenses made from glass differs strongly enough to necessitate a detailed optical, and possibly an economic analysis.

Note the difference between the ideal nonimaging concentration ratio $n/\sin\theta$, derived in Sect. 3, where n is the refractive index of the material between the thin lens and the absorber, here $n_{\mathrm{air}} = 1.0$; and the concentration ratio $1/\sin\theta$ referring to the refractive index of the thin lens itself. We find that two refractive indices have an influence on the concentration ratio of the nonimaging lens,

- the refractive index of the thin lens itself, fulfilling the conditions of maximum concentration $1/\sin\theta$ for $n \to \infty$. The concentration ratio of the real lens ($n \approx 1.5$) is discriminated by about 20–25% when compared to the ideal lens of $n \to \infty$;
- the refractive index of the translucent material filling the gap in between thin lens and absorber; leading to the familiar ideal concentration ratio $n/\sin\theta$.

Unfortunately, we are not able to further examine the performance of lenses of higher refractive index, because the wavelength-dependent dispersive properties of these materials are not available, even though the refractive power of the material is given. Diamonds have very strong dispersive power, and properly cut, let colors sparkle around those whom they are decorating. Strong dispersion may make them unsuitable for concentrating wavelengths across the solar spectrum. At the ideal lens concentrator, refractive indices for all wavelengths must be $n(\lambda) = \infty$.

Table 1. Geometrical concentration ratios C_{g} for lenses of different refractive indices n_{D}, and their comparison to the ideal $C = 1/\sin\theta$. Lenses of $\theta = \pm 1°$ and $\psi = \pm 6°$.

Material	n_{D}	C_{g}	$C_{\mathrm{g}}/C_{\mathrm{ideal}}$
Air-like	1.01	6.3	0.11
Water	1.32	36.3	0.63
PMMA	1.49	42.5	0.74
Glass	1.68	46.5	0.81
Diamond	2.42	52.8	0.92
Ideal[a]	50.0	56.2	0.98

[a] The refractive index $n = 50.0$ is taken in our simulation as representing $n \to \infty$. Due to its numerical approach, the prisms can be designed by the program only in accordance to a confidence level, here $\Delta E = 10^{-6}$.

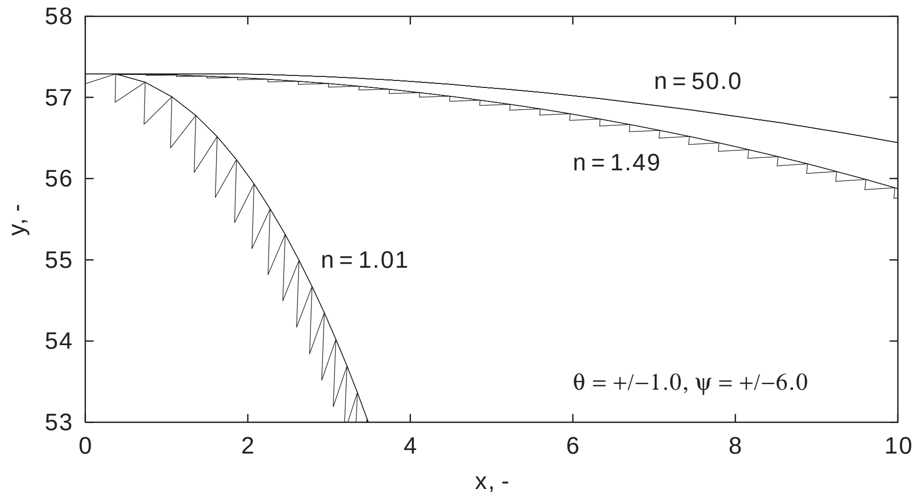

Figure 7. Prism angles and inclinations at prisms of lenses of three refractive indices

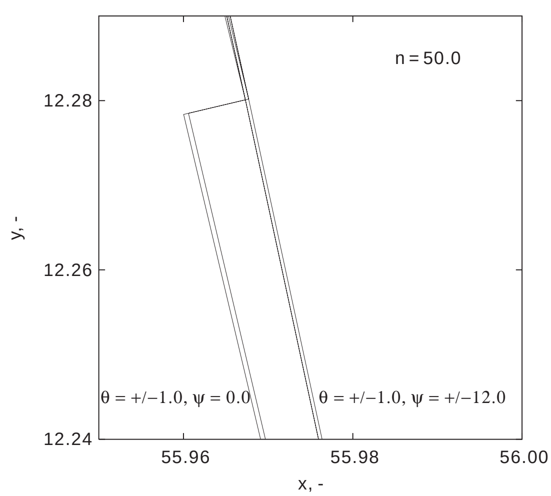

Figure 8. Prisms of the ideal nonimaging Fresnel lens with refractive index $n \to \infty$, small but existing influence of the perpendicular acceptance half angle ψ on lens width and prism angle

Prism angles and inclinations are given enlarged in Fig. 7, where it is clearly shown that prisms with low refractive index have to substitute this lack in refractive power with a large prism angle β which can only be sustained with a steep slope of the prism front.

Figure 8 presents some of the outermost prisms of two lenses of with different secondary acceptance half angle ψ. The prism angles are small, while the prism inclinations are extremely negative, which is facilitated by the high refractive index.

3.2. Design angle ψ

The incorporation of the perpendicular acceptance half angle ψ into the design of the nonimaging Fresnel lens does change the shape of the ideal linear lens, as seen in Fig. 8. Contrary to the 2D-lens, the three-dimensional Fresnel lens concentrator ($\psi = 0$) is not influenced by any effects of the perpendicular design angle.

At the linear lens, incidence from $0 < \psi_{\text{in}} \leq \psi$ is considered in the three-dimensional design process. If $\psi \neq 0$, and $\theta =$ *constant*, the absorber is not covered by the designed flux. Focal shortening happens in both the cross-sectional, and in the perpendicular direction of incidence. When the perpendicular incidence angle is smaller than

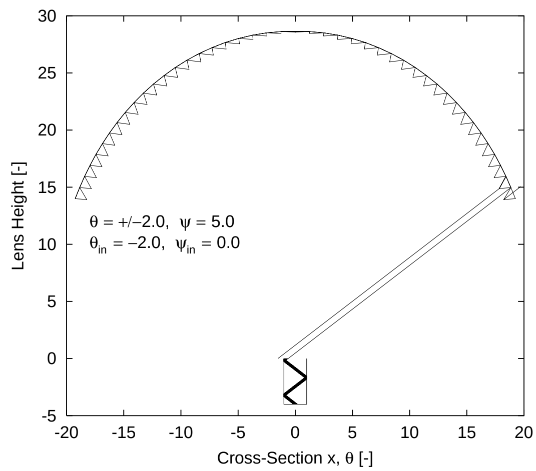

Figure 9. Nonimaging Fresnel lens with kaleidoskope diffusor; ray tracing for one ray bundle incident at $\theta_{\text{in}} = -2.0$ degrees from the right, and $\psi_{\text{in}} = 0.0$ out of the plane of the paper. The lens has design acceptance half angles of θ and ψ

the perpendicular acceptance half angle, the focal plane moves from a position found for the perpendicular design angle further down below the lens. For a prism located in the right part of the lens, some rays incident at angles smaller than ψ are missing the absorber on its right side.

3.3. Total reflection and truncation

Total internal reflection may occur for lower prisms, effectively limiting the width of the lens due to the resulting minimum required prism inclination. This is a design inherent problem and limits the performance of the first aperture. While total internal reflection is related to the limit of concentration of the ideal lens, near total reflection on the outer surface of the lens for prisms towards the absorber level is a problem limiting the performance of the practical lens.

For the ideal nonimaging concentrator, the exit aperture angle must be 90°. This is impossible with a truncated lens, which is cut at some height over the absorber, unless secondary concentration is employed. The truncated lens cannot be an ideal concentrator.

4. FIXING THE FLUX DISTRIBUTION

Solar spectral reproduction can be problematic for the optimum performance of a photovoltaic multijunction device. Possible solutions include the following[1]:

- The redesign of the nonimaging lens: in theory, the nonimaging lens can be designed to produce uniform flux. The argument is that the degree of freedom remaining between the ideal theoretical lens with its uniform flux and the nonideal practical lens could be used to create a lens with prescribed flux distribution. Related work is under way by several researchers, and definite results are expected at this very conference.
- The movement of the absorber closer to the lens or further away from it: A closer absorber performs marginally better in terms of color reproduction, but the absorber misses increase. In the case of the absorber being further away from the lens, less absorber misses are recorded, but worse color reproduction is observed. For an absorber located at $1.05f$, color reproduction at the center of the flux is quite good, while the system becomes more sensitive to incidence angles off normal, and flux uniformity over the absorber is strongly reduced, showing a strong peak.
- The placement of a reflective or refractive secondary concentrator in place of the original receiver: the secondary concentrator is called a homogenizer if the aim is to control the flux adding little or no geometrical concentration.

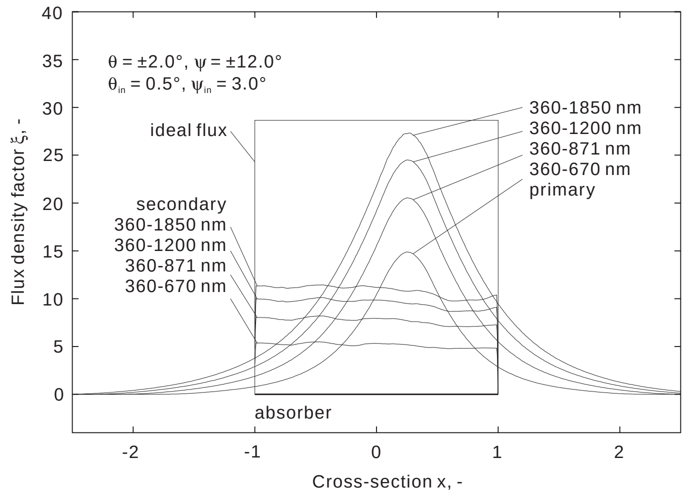

Figure 10. Color flux density factors and solar spectral reproduction for the nonimaging Fresnel lens of acceptance half-angle pairs $\theta = \pm 2°$ and $\psi = \pm 12°$ with and without secondary kaleidoskope-based homogenizer. Incidence away from the normal by $\theta_{\rm in} = 0.5°$ and $\psi_{\rm in} = 3.0°$. Depth of the secondary $12.6\,d$

The homogenizer can be an option to redirect rays and make the color flux more uniform, but first-order reflection losses at the secondary are substantial, and may exceed the losses caused by inhomogeneous flux. The prism sheet secondary[7,8] is used to refract rays away from the grid on the surface of a photovoltaic cell to avoid shading losses.

A kaleidoskope-based secondary homogenizer[9,10] for the linear lens can be constructed from two parallel mirrors forming a trough of absorber width under the original absorber level (Fig. 9). In the example given in Fig. 10 (inspired by J. Gordon), we use a secondary with a depth of $12.6\,d$, where d is the absorber half-width. Clearly, the flux uniformity and the reproduction of the solar spectrum improve. Additional reflection losses can be calculated with

$$\tau_{\rm CPC} = \rho_{\rm m}^{\rm n} \ . \tag{7}$$

With (7), reflection losses at the mirror sides of the secondary are 9.8% using a reflectivity of the mirror $\rho_{\rm m} = 0.95$ and the average number of reflections $n = 2.01$. The effectiveness of the kaleidoskope-based secondary was confirmed in our experiment.

- The design of photovoltaic cells (expected to be the main application of the nonimaging lens): instead of the redesign of the concentrator, the layers of the multijunction device could be designed and assembled with varying thickness according to the spectral fraction incident at that point. This may not be practical, since the angle of incidence of the radiation on the concentrator has a strong influence on the flux distribution at the absorber. Still, multijunction devices which are to be used with refractive solar concentrators must be designed for the flux and spectral reproduction the lens produces, i.e. the spectral transmittance of the lens material is a design parameter.

These measures not only complicate the design of the concentrator, but may also carry the risk of lower performance at incidence angles off normal. Careful analysis and design are necessary.

5. CONCLUSIONS

In conclusion, to qualify as ideal, the nonimaging Fresnel lens must be

- a Lambertian radiator, with refractive index of infinity for all incident wavelengths; it follows that the lens has an optimum elliptical shape with smooth outer surface;

- a 3D-concentrator ($\psi = 0$), as to eliminate the losses induced by the perpendicular acceptance half angle;

We note that the relations between the concentrators' dimensions (2D or 3D), and their ideal behavior is reversed for the CPC and for the nonimaging Fresnel lens. The CPC is ideal as two-dimensional concentrator; but when designed in rotational symmetry, some skew rays within the acceptance half angle are rejected.[4] The nonimaging Fresnel lens is ideal as three-dimensional concentrator, when there is no influence of the perpendicular acceptance half angle.

The fundamental difference between CPC and nonimaging Fresnel lens is that, while an ideal CPC can actually be constructed, the ideal nonimaging lens cannot, as a material with refractive index approaching infinity does not exist.

Total reflection, internal and on the outer surface, as well as the prism angle and inclination are determined by the refractive index of the prism material, and effectively limit the width of the lens. Truncation further deteriorates the degree of ideal performance of the lens.

A simple secondary concentrator, the kaleidoskope-based flux homogenizer, can be used to create uniform flux, including the reproduction of the spectral characteristics of the incoming light (here of the solar spectrum). Additional reflection losses of approximately 10% can be justified, if the application requires accurate flux uniformity, e.g. in multijunction solar cells.

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