Thermal Stress Pre-Analysis

First, we need to know if the expansion of the bar is greater than the free space between the bar and the wall $x = .002$ meters

$$\delta_T = \alpha L \Delta T \tag{1}$$

If $\delta_T > x$, then we know there will be stress on the bar. Otherwise, the stress on the bar $\sigma_{xx} = 0$ Inserting the values, we find

$$\delta_T = (1.2 \times 10^{-5} \ {}^\circ C^{-1})(5 \text{ m})(100 \ {}^\circ C) = .006 \text{ m} > .002 \text{ m} \tag{2}$$

Now that we know the deformation due to the change in temperature will be greater than the space between the wall and the bar, we know that there will be a stress on the bar. We can observe from the problem that if the wall was not there, the bar would deform δ_T. However, because the wall is there, the bar will not deform the full δ_T We can further break up the problem into this equation

$$\delta_T = x + \delta_\sigma \tag{3}$$

Where δ_T is the strain contribution from the change in temperature and δ_σ is the strain contribution from the force imparted by the wall on the bar. Therefore

$$\alpha L \Delta T = x + \frac{PL}{EA} \tag{4}$$

substituting $\sigma_{xx} = \frac{P}{A}$, we get a solvable eqaution

$$\alpha L \Delta T = x + \frac{\sigma_{xx} L}{E} \tag{5}$$

Solving for σ, we find that

$$\sigma_{xx} = \alpha E \Delta T - \frac{xE}{L} \tag{6}$$

After substituting, we find

$$\sigma_{xx} = (1.2 \times 10^{-5} \ {}^\circ C^{-1})(2 \times 10^{11} \text{ Pa})(100 \ {}^\circ C) - \frac{.002 \text{ m}(2 \times 10^{11} \text{ Pa})}{5 \text{ m}} \tag{7}$$

$$\sigma_{xx} = 160 \text{ MPa} \tag{8}$$