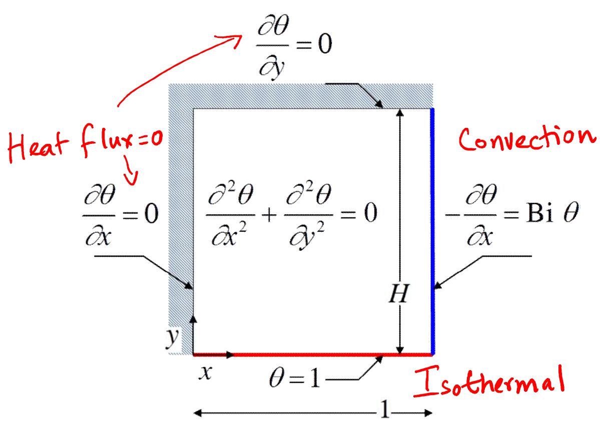
2D Steady Heat Conduction in a Block: Problem Overview

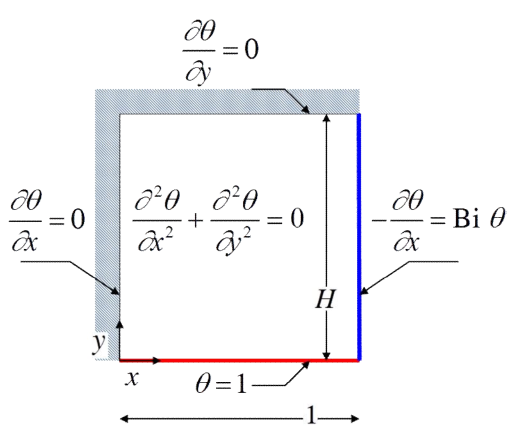


Find temperature and heat flux distribution

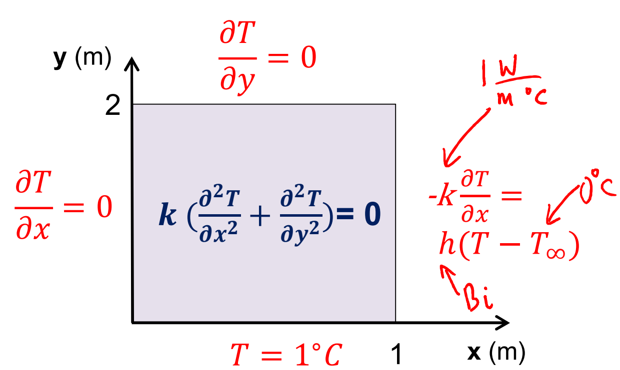
- Dimensionless temperature: $\theta \equiv \frac{T - T_\infty}{T_o - T_\infty}$
- T_o: Constant temperature of bottom face
- T_∞: Far temperature of fluid bathing the right face
- $\frac{H}{W} = 2, Bi = h\frac{w}{k} = 5$ (Biot number)

Mathematical Model: Boundary Value Problem

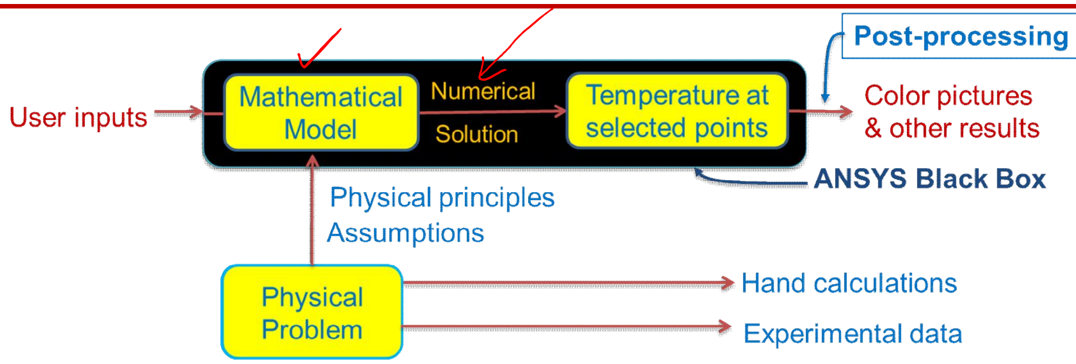
Non-dimensional BVP



Dimensional BVP in ANSYS



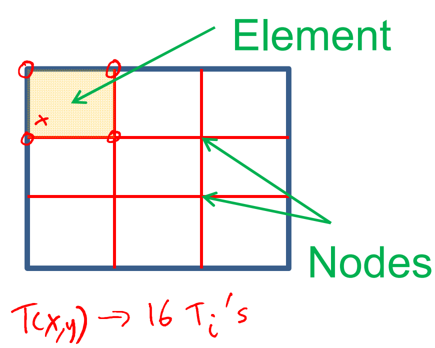
Pre-Analysis



- Mathematical model
- Numerical solution strategy
- Hand-calculations of expected results/trends

Numerical Solution Strategy: Finite-Element Method

- Divide domain into elements and nodes
- Reduce the problem to determining temperature values only at nodes (T_i)
  - Use polynomial interpolation to relate temperature within an element to $T_i's$



How to Find Nodal Temperatures T_i?

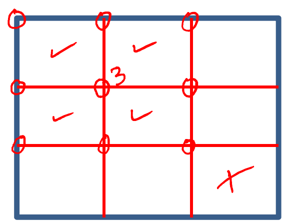
Mathematical Model (Boundary Value Problem) → ? → System of algebraic equations in nodal temperatures → Invert → Nodal temperatures T_i → Post processing → $T(x,y),\ \vec{q}(x,y)$

Piecewise polynomial approximation for T

$[K]\{T\} = \{Q\}$

[16x16] {16x1}

Each algebraic equation will relate a nodal temperature to its neighbors



How to Derive System of Algebraic Equations?

Piecewise polynomial approximation for T

$\nabla^2 T = 0$ → ~~System of algebraic eqs. in nodal temperatures~~

$\nabla^2 T = 0$ → Weighted Integral Form

$\int w(\nabla^2 T)\, dxdy = 0$

$w(x,y)$ is an arbitrary function

→ $\int w^e(\nabla^2 T) dxdy = 0$

w^e(x,y) is an arbitrary piecewise polynomial function

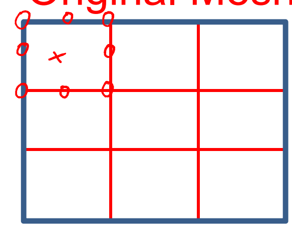
Piecewise polynomial approximation for T & W

System of algebraic eqs. in nodal temperatures

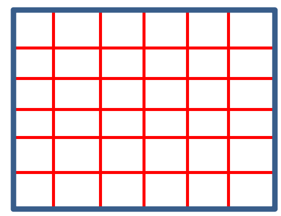
How to Reduce the Error in the Finite Element Solution?

- FE solution tends to the exact solution as polynomial approximation gets better
- Two ways to do this
  - Increase no. of elements
  - Increase order of polynomial within each element
    - Use more nodes per element

Original Mesh

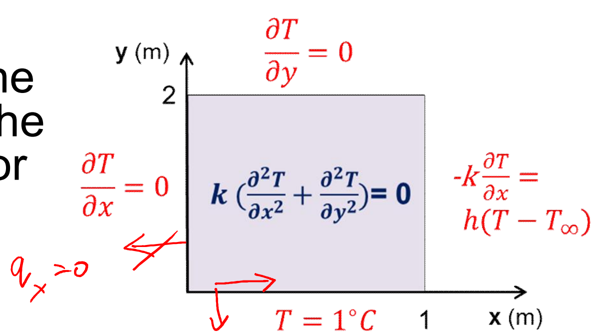


Refined Mesh



Hand Calculations: Predict Expected Results/Trends

- What is the direction of the heat flow at the boundaries for $\mathrm{Bi} = \frac{hw}{k} = 5$?



Hand Calculations: Analytical Solution (courtesy Prof. Michel Louge)

$$\theta = \sum_{j=1}^{\infty} f_j \theta_j(x, y),$$

$$f_j = \frac{2 \sin \lambda_j}{\lambda_j [1 + \exp(2\lambda_j H^*)]} \Big/ \left[1 + \frac{\mathrm{Bi}}{\mathrm{Bi}^2 + \lambda_j^2}\right],$$

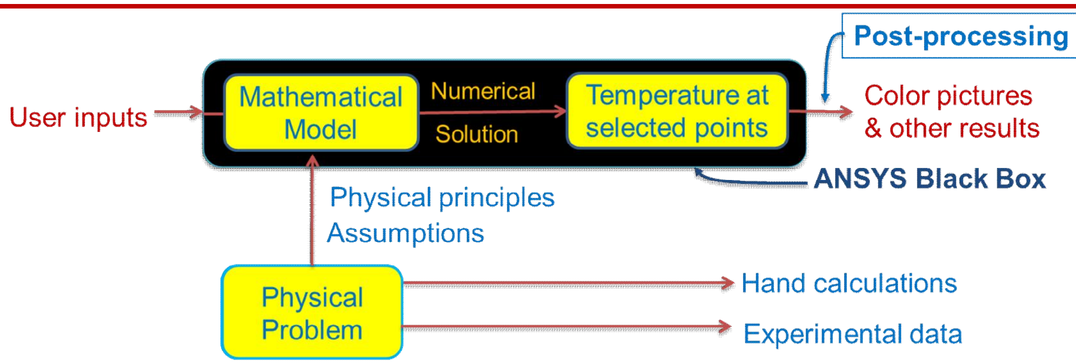
The eigensolutions are

$$\theta_j = \cos(\lambda_j x^*) \exp(\lambda_j y^*) \{1 + \exp[2\lambda_j (H^* - y^*)]\},$$

with eigenvalues satisfying

$$\lambda_j \tan \lambda_j = \mathrm{Bi}.$$

Verification & Validation



- Verification: Did I solve the model right?
  - Check consistency with mathematical model, level of numerical errors, comparison with hand calcs
- Validation: Did I solve the right model?
  - Check against experimental data

Verification Steps

1. Sanity checks
2. Does the FEA solution honor the boundary conditions in the mathematical model?
3. Does the FEA solution honor the physical principles in the mathematical model?
   – Check energy conservation in the domain
4. Is the discretization error acceptable?
   – Perform mesh refinement studies
5. Does the FEA solution match the analytical solution?