PSS TEAM - FALL 2010 - Appendix - Definitions, Equations, and some values

This appendix summarizes the definitions and equations used by the PSS team. We also included some values used in our experiments. This file is not the file we are using for performing our calculations, eventhough the equations are copied from our calculations worksheets.

Various physical definitions relevant to the process and the first experiment

Some values

These are characteristics of both the flocs and chemical compounds relevant to the Plate Settler Spacing Team's flocculation process. These values are used to calibrate our experiments.

$d_0 := 1\mu m$

$MW_{Hydrogen} := 1.008 \frac{gm}{mol}$

$d_{Final} := 1mm$

$MW_{Ca} := 40 \frac{gm}{mol}$

$d_{fractal} := 2.3$

$MW_{Sulfur} := 32.064 \frac{gm}{mol}$

$C_{Clay} := 100 \frac{mg}{L}$

$MW_{Al} := 27 \frac{gm}{mol}$

$C_{Alum} := 45 \frac{mg}{L}$

$MW_O := 16 \frac{gm}{mol}$

$Carbon := 12 \frac{gm}{mol}$

$MW_{CalciumCarbon} := MW_{Ca} + Carbon + 3MW_O$

$\rho_{AlOH3} := 2420 \frac{kg}{m^3}$

$MW_{Alum} := 2 \cdot MW_{Al} + 3\left(MW_{Sulfur} + 4 \cdot MW_O\right) + 14.3\left(2 \cdot MW_{Hydrogen} + MW_O\right)$

$\nu := 1 \cdot 10^{-6} \frac{m^2}{s}$

$MW_{AlOH3} := MW_{Al} + 3\left(MW_O + MW_{Hydrogen}\right)$

$\rho_{H2O} := 1000 \frac{kg}{m^3}$

$\rho_{Clay} := 2650 \frac{kg}{m^3}$

$\mu_{H2O} := 1.002 \cdot 10^{-3} \frac{kg}{m \cdot s}$

$D_{Fractal} := 2.3$

$\phi := \frac{45}{24}$ is a sphericity factor to take into account that the floc particles are not exactly solid spheres.

$$C_{AlOH3}\left(C_{Alum}\right) := \frac{C_{Alum}}{MW_{Alum}} \cdot 2MW_{AlOH3}$$

$$C_{Floc}\left(C_{Alum}, C_{Clay}\right) := C_{AlOH3}\left(C_{Alum}\right) + C_{Clay}$$

$$\phi_{Floc.0}\left(C_{Alum}, C_{Clay}\right) := \frac{C_{AlOH3}\left(C_{Alum}\right)}{\rho_{AlOH3}} + \frac{C_{Clay}}{\rho_{Clay}}$$

$$\rho_{Floc.0}\left(C_{Alum}, C_{Clay}\right) := \frac{C_{Floc}\left(C_{Alum}, C_{Clay}\right)}{\phi_{Floc.0}\left(C_{Alum}, C_{Clay}\right)}$$

Characteristics of the plate/tube settlers experiments and simulations

Because we want to measure the effect of the velocity gradient, we set $v_{capture} := 10\frac{m}{day}$ for all experiments.

$\alpha := 60deg$ is the inclination of the tube/plate settlers relative to the normal

$V_{\alpha}(v_{up}) := \frac{v_{up}}{\sin(\alpha)}$ is the velocity through the tube settler taking into account its sixty degree angle relative to the normal.

$tube := "tube"$ $\quad plate := "plate"$

$$V_{ratio}(geometry) := \begin{vmatrix} 1.5 \text{ if } geometry = plate \\ 2 \text{ if } geometry = tube \end{vmatrix}$$

This V_{ratio} is a geometric factor that accounts for the change in the velicity profile between tubes and plates settlers

For the PSS Team's first set of experiments, it tests different upflow velocities through the tube settlers: $v_{up1} := 1\frac{mm}{s}$, $v_{up2} := 2\frac{mm}{s}$, $v_{up3} := 5\frac{mm}{s}$ and some experiments should run at $v_{up4} := 9\frac{mm}{s}$. These similarly correspond to different velocity gradients, which can be found later on in the file.

$n_{tube} := 1$

$d_{tube1} := 25.4mm$ $\quad d_{tube2} := 22.225mm$ $\quad d_{tube3} := 15.875mm$

$d_{tube4} := 12.7mm$ $\quad d_{tube5} := 9.53mm$ $\quad d_{tube6} := 6.35mm$

The number of tubes being used. As a control parameter this is kept at one.

These are the available diameters that the current Plate Settler Spacing Team had at its disposal for the Spring 2010 and Fall 2010 semesters.

Testing tubes 3-6, corresponding to 5/8", 1/2", 3/8", 1/4"

Plate and tube settlers calculations

$$d_{floc}(v_{capture}) := d_0 \cdot \left[\left(\frac{18 \cdot \phi \cdot v_{capture} \cdot \nu}{g \cdot \sin(\alpha) \cdot d_0^{\,2}} \right) \cdot \frac{\rho_{H2O}}{\rho_{Floc.0}(C_{Alum}, C_{Clay}) - \rho_{H2O}} \right]^{\frac{1}{d_{fractal}-1}}$$

This is the smallest floc that can be captured reliably with a capture velocity of 10 m/day or 0.116 mm/s.

$d_{floc}(v_{capture}) = 76.962 \cdot \mu m$

An equation for the terminal velocity of a floc given its diameter:

$$V_t(C_{Alum}, C_{Clay}, d_{fractal}, d_0, d_{floc}) := \frac{g \cdot \sin(\alpha) \cdot {d_0}^2}{18 \cdot \phi \cdot \nu} \cdot \frac{\rho_{Floc.0}(C_{Alum}, C_{Clay}) - \rho_{H2O}}{\rho_{H2O}} \cdot \left(\frac{d_{floc}(v_{capture})}{d_0}\right)^{d_{fractal}-1}$$

$$V_t(C_{Alum}, C_{Clay}, d_{fractal}, d_0, d_{floc}) = 10 \cdot \frac{m}{day}$$

Functional definitions

These are functions which are used to calculate the physical parameters of the tube settler system for subsequent experiments.

THE PI RATIO

For the time being, let's set $V_{Up} := v_{up1}$ as our reference and $d_{tube} := d_{tube1} = 1\ in$. And we assume that $d_{Floc} := d_0, 2 \cdot d_0 .. d_{Final}$

This is the "pi" ratio established from last semester's file. It is the ratio of the terminal velocity of the floc to the velocity experienced at its outer diameter. When this value is less than one, floc roll-up should occur.

$$V_{particleexperice}(V_{Up}, d_{Tube}, d_{Floc}, geometry) := V_{ratio}(geometry)\ V_\alpha(V_{Up}) \cdot \left[1 - \left(\frac{\frac{d_{Tube}}{2} - d_{Floc}}{\frac{d_{Tube}}{2}}\right)^2\right]$$

$$\Pi_{V.Definition}(C_{Alum}, C_{Clay}, d_{fractal}, d_0, d_{tube}, v_{capture}, v_{up}) := \frac{V_t(C_{Alum}, C_{Clay}, d_{fractal}, d_0, d_{floc})}{V_{particleexperice}(V_{Up}, d_{tube}, d_{Floc}, tube)}$$

$$\Pi_{V.Developed1}(C_{Alum}, C_{Clay}, d_{fractal}, d_0, d_{tube}, v_{capture}, v_{up}) := \frac{V_t(C_{Alum}, C_{Clay}, d_{fractal}, d_0, d_{floc})}{2\ V_\alpha(v_{up}) \cdot \left[1 - \left(\frac{\frac{d_{tube}}{2} - d_{floc}(v_{capture})}{\frac{d_{tube}}{2}}\right)^2\right]}$$

$$\Pi_{V.Developed2}(C_{Alum}, C_{Clay}, d_{fractal}, d_0, d_{tube}, v_{capture}, v_{up}) := \frac{\frac{g \cdot \sin(\alpha) \cdot {d_0}^2}{18 \cdot \phi \cdot \nu} \cdot \frac{\rho_{Floc.0}(C_{Alum}, C_{Clay}) - \rho_{H2O}}{\rho_{H2O}} \cdot \left(\frac{d_{Floc}}{d_0}\right)^{D_{Fractal}-1}}{2\ V_\alpha(v_{up}) \cdot \left[1 - \left(\frac{\frac{d_{tube}}{2} - d_{floc}(v_{capture})}{\frac{d_{tube}}{2}}\right)^2\right]}$$

OTHER USEFUL EQUATIONS

This is an equation to calculate the length of tubing neeed given a specific capture velocity, diameter, and upflow velocity:

$$L_{tube}(\alpha, v_{capture}, d_{tube}, v_{up}) := \left(\frac{V_{\alpha}(v_{up})\cdot\sin(\alpha)}{v_{capture}} - \sin(\alpha)^2\right)\cdot\frac{d_{tube}}{\sin(\alpha)\cdot\cos(\alpha)}$$

$$L_{tube}(\alpha, v_{capture}, d_{tube3}, V_{Up}) = 0.289\,m$$

Calculation of the flow rate through the tube, given a fixed length, diameter and capt ure velocity.

$$Q_{tube}(d_{tube}, L_{tube}, v_{capture}) := \left(\frac{L_{tube}}{d_{tube}}\cos(\alpha)\sin(\alpha) + \sin(\alpha)^2\right)\cdot\pi\cdot\frac{(d_{tube})^2}{4}\cdot v_{capture}$$

Velocity in the tube as a function of the distance from the wall, where the floc diameter is this distance.

$$v_{tube}(r, Q_{tube}, d_{tube}) := \frac{2\cdot Q_{tube}}{\pi\cdot\left(\frac{d_{tube}}{2}\right)^4}\left[\left(\frac{d_{tube}}{2}\right)^2 - r^2\right]$$

This is the linearized velocity gradient.

$$v_{gradient}(v_{\alpha}, d_{tube}) := \frac{4\cdot v_{\alpha}}{\left(\frac{d_{tube}}{2}\right)}$$

A function to solve for the upflow velocity in a tube.

$$v_{Up}(d_{tube}, L_{tube}, v_{capture}) := \frac{Q_{tube}(d_{tube}, L_{tube}, v_{capture})}{\left[\frac{\pi(d_{tube})^2}{4}\right]}$$