

# Pre-Analysis

## 1 $\sigma_x$

First, let's begin by finding the average stress, the nominal area stress, and the maximum stress with a concentration factor:

$$\sigma_o = \frac{F}{A} = \frac{100000 \text{ lb}}{.1 \times 5 \text{ in}^2} = 2 \times 10^5 \text{ psi}$$

$$\sigma_{nominal} = \frac{F}{A} = \frac{100000 \text{ lb}}{.1 \times (5 - .5) \text{ in}^2} = 2.222 \times 10^5 \text{ psi}$$

Using a Concentration Factor Chart (3250 Students: See Figure 4.22 on page 158 in *Deformable Bodies and Their Material Behavior*), we find that  $\frac{d}{w} = .1$  and thus  $K \approx 2.73$ . Now we can find the maximum stress using the nominal stress and the concentration factor

$$\sigma_{max} = K \times \sigma_{nominal} = (2.73)(2.222 \times 10^5 \text{ psi}) = 6.0667 \times 10^5 \text{ psi}$$

## 2 $\sigma_r$

Now, let's look at the radial stress varies in the plate:

$$\sigma_r(r, \theta) = \frac{1}{2}\sigma_o\left[\left(1 - \frac{a^2}{r^2}\right) + \left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right)\cos(2\theta)\right]$$

assuming  $r \gg a$

$$\sigma_r(r, \theta) = \sigma_r(\theta) = \frac{1}{2}\sigma_o[1 + \cos(2\theta)]$$

Now we will examine the stress far from the hole at  $\theta = 0$  and  $\frac{\pi}{2}$

$$\sigma_r(0) = \frac{1}{2}\sigma_o[1 + \cos(2(0))] = \sigma_o$$

$$\sigma_r\left(\frac{\pi}{2}\right) = \frac{1}{2}\sigma_o[1 + \cos(2\left(\frac{\pi}{2}\right))] = 0$$

### 3 $\sigma_\theta$

Now we will examine how  $\sigma_\theta$  varies in the plate. We will approach this very similarly to how we approached the examination of  $\sigma_r$ :

$$\sigma_\theta(r, \theta) = \frac{1}{2}\sigma_o\left[\left(1 + \frac{a^2}{r^2}\right) - \left(1 + 3\frac{a^4}{r^4}\right)\cos(2\theta)\right]$$

assuming  $r \gg a$

$$\sigma_\theta(r, \theta) = \sigma_\theta(\theta) = \frac{1}{2}\sigma_o[1 - \cos(2\theta)]$$

Now we will examine the stress far from the hole at  $\theta = 0$  and  $\frac{\pi}{2}$

$$\sigma_\theta(0) = \frac{1}{2}\sigma_o[1 - \cos(2(0))] = 0$$

$$\sigma_\theta\left(\frac{\pi}{2}\right) = \frac{1}{2}\sigma_o[1 - \cos(2\left(\frac{\pi}{2}\right))] = \sigma_o$$

### 4 $\tau_{r\theta}$

Finally, we will examine how the shear stress in the  $r\theta$  direction varies in the plate. The equation for the shear stress in the plate is:

$$\tau_{r\theta} = -\frac{1}{2}\sigma_o\left(1 - 3\frac{a^4}{r^4} + 2\frac{a^2}{r^2}\right)\sin(2\theta)$$

assuming  $r \gg a$

$$\tau_{r\theta}(r, \theta) = \tau_{r\theta}(\theta) = -\frac{1}{2}\sigma_o\sin(2\theta)$$

Now we will examine the values of  $\tau_{r\theta}$  when  $r \gg a$  and at  $\theta = 0$  and  $\theta = \frac{\pi}{2}$

$$\tau_{r\theta}(0) = -\frac{1}{2}\sigma_o\sin(2(0)) = 0$$

and

$$\tau_{r\theta}\left(\frac{\pi}{2}\right) = -\frac{1}{2}\sigma_o\sin(2\left(\frac{\pi}{2}\right)) = 0$$

We will reexamine all of these calculations so we may estimate the validity of the ANSYS simulation later in this tutorial.