

Pre-Analysis

1 σ_x

First, let's begin by finding the average stress, the nominal area stress, and the maximum stress with a concentration factor.

$$\sigma_o = \frac{F}{A} = \frac{100000 \text{ lb}}{.2 \times 5 \text{ in}^2} = 1 \times 10^6 \text{ psi}$$

$$\sigma_{nominal} = \frac{F}{A} = \frac{100000 \text{ lb}}{.2 \times (5 - .5) \text{ in}^2} = 1.111 \times 10^6 \text{ psi}$$

For a finite plate with a hole, there is no analytical solution. However, an analytical solution does exist for an infinite plate with a hole. The concentration factor for an infinite plate with a hole is $K = 3$. The maximum stress for an infinite plate with a hole is

$$\sigma_{max} = K \times \sigma_o$$

$$\sigma_{max} = (3.0)(1.0 \times 10^6 \text{ psi}) = 3.0 \times 10^6 \text{ psi}$$

Although there is no analytical solution for a finite plate with a hole, there is empirical data available to find a concentration factor. Using a Concentration Factor Chart (3250 Students: See Figure 4.22 on page 158 in *Deformable Bodies and Their Material Behavior*), we find that $\frac{d}{w} = .1$ and thus $K \approx 2.73$. Now we can find the maximum stress using the nominal stress and the concentration factor

$$\sigma_{max} = K \times \sigma_{nominal} = (2.73)(1.111 \times 10^6 \text{ psi}) = 3.033 \times 10^6 \text{ psi}$$

2 σ_r

Now, let's look at the radial stress varies in the plate:

$$\sigma_r(r, \theta) = \frac{1}{2}\sigma_o\left[\left(1 - \frac{a^2}{r^2}\right) + \left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right)\cos(2\theta)\right]$$

at $r=a$

$$\sigma_r = 0$$

assuming $r \gg a$

$$\sigma_r(r, \theta) = \sigma_r(\theta) = \frac{1}{2}\sigma_o[1 + \cos(2\theta)]$$

Now we will examine the stress far from the hole at $\theta = 0$ (the x-axis) and $\frac{\pi}{2}$ (the y-axis)

$$\sigma_r(0) = \frac{1}{2}\sigma_o[1 + \cos(2(0))] = \sigma_o$$

$$\sigma_r\left(\frac{\pi}{2}\right) = \frac{1}{2}\sigma_o[1 + \cos(2\left(\frac{\pi}{2}\right))] = 0$$

3 σ_θ

Now we will examine how σ_θ varies in the plate. We will approach this very similarly to how we approached the examination of σ_r :

$$\sigma_\theta(r, \theta) = \frac{1}{2}\sigma_o\left[\left(1 + \frac{a^2}{r^2}\right) - \left(1 + 3\frac{a^4}{r^4}\right)\cos(2\theta)\right]$$

at $r = a$

$$\sigma_\theta = \frac{1}{2}\sigma_o(2 - 4\cos(2\theta))$$

assuming $r \gg a$

$$\sigma_\theta(r, \theta) = \sigma_\theta(\theta) = \frac{1}{2}\sigma_o[1 - \cos(2\theta)]$$

Now we will examine the stress far from the hole at $\theta = 0$ and $\frac{\pi}{2}$

$$\sigma_\theta(0) = \frac{1}{2}\sigma_o[1 - \cos(2(0))] = 0$$

$$\sigma_\theta\left(\frac{\pi}{2}\right) = \frac{1}{2}\sigma_o[1 - \cos(2\left(\frac{\pi}{2}\right))] = \sigma_o$$

4 $\tau_{r\theta}$

Finally, we will examine how the shear stress in the $r\theta$ direction varies in the plate. The equation for the shear stress in the plate is:

$$\tau_{r\theta} = -\frac{1}{2}\sigma_o\left(1 - 3\frac{a^4}{r^4} + 2\frac{a^2}{r^2}\right)\sin(2\theta)$$

at $r = a$

$$\tau_{r\theta} = 0$$

assuming $r \gg a$

$$\tau_{r\theta}(r, \theta) = \tau_{r\theta}(\theta) = -\frac{1}{2}\sigma_o \sin(2\theta)$$

Now we will examine the values of $\tau_{r\theta}$ when $r \gg a$ and at $\theta = 0$ and $\theta = \frac{\pi}{2}$

$$\tau_{r\theta}(0) = -\frac{1}{2}\sigma_o \sin(2(0)) = 0$$

and

$$\tau_{r\theta}\left(\frac{\pi}{2}\right) = -\frac{1}{2}\sigma_o \sin\left(2\left(\frac{\pi}{2}\right)\right) = 0$$

We will reexamine all of these calculations so we may estimate the validity of the ANSYS simulation later in this tutorial.