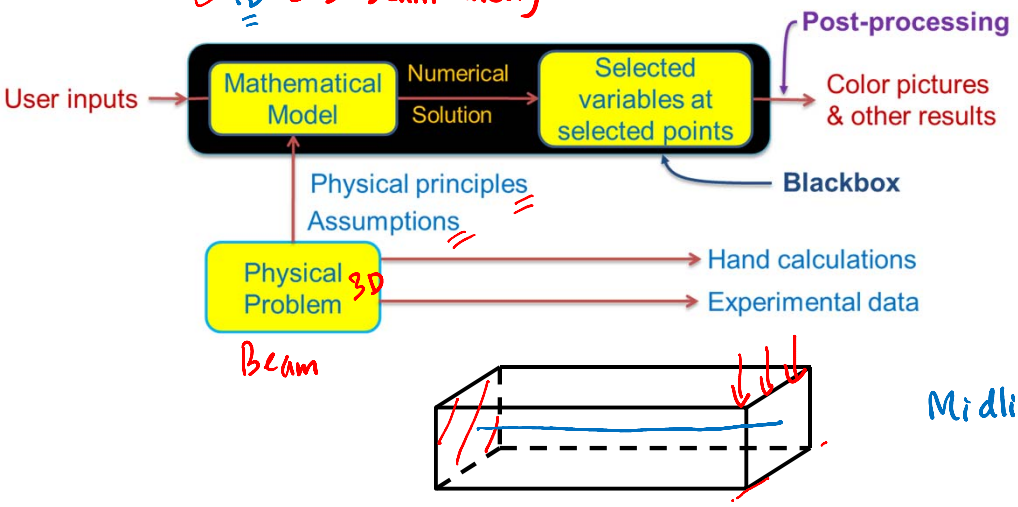
What's Under the Blackbox?

Cornell Engineering

3D elasticity

2D elasticity

1D E-B beam theory



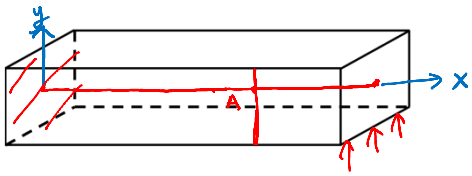
Beam

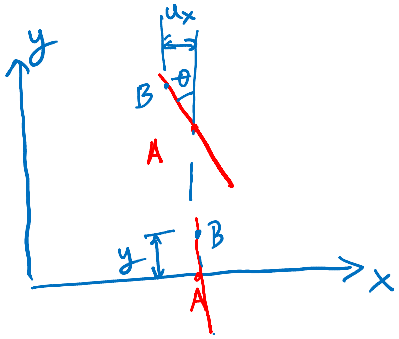
Midline

Euler-Bernoulli Beam Theory

Timoshenko beam theory

Cornell Engineering





$$u_x = -y\sin\theta \simeq -y\theta = -y\frac{du_y}{dx}$$

$$u_y \simeq u_y(x)$$

Assumptions

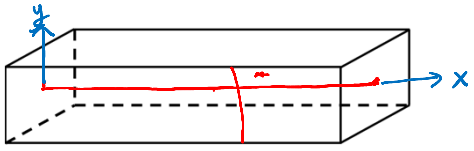
No axial load

Plane sections remain plane

Small θ

Strains and Stresses

Cornell Engineering



$u_x \simeq -y \frac{du_y}{dx}$

$u_y \simeq u_y(x)$

$\varepsilon_x = \frac{\partial u_x}{\partial x} = -y \frac{d^2 u_y}{dx^2}$

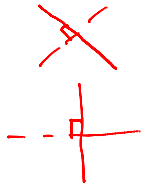
$\varepsilon_y = \frac{\partial u_y}{\partial y} = 0$

$\gamma_{xy} = 0$

$\sigma_x = E \varepsilon_x$

$\sigma_y = 0$

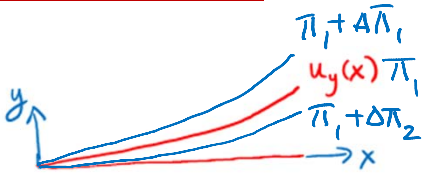
$\tau_{xy} = 0$



Potential Energy, Π

Cornell Engineering

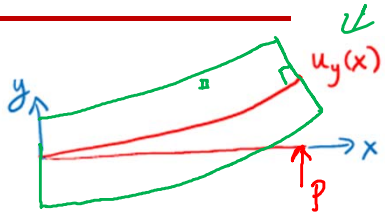
$u_y(x) \rightarrow \begin{matrix} u_x(x,y) \\ u_y(x,y) \end{matrix} \rightarrow \begin{matrix} \varepsilon_x \\ \sigma_x \end{matrix} \rightarrow \Pi$ Minimize

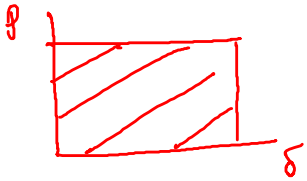


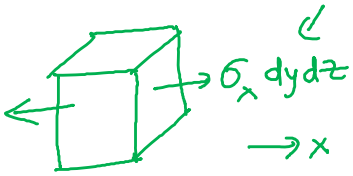
Potential Energy Minimization

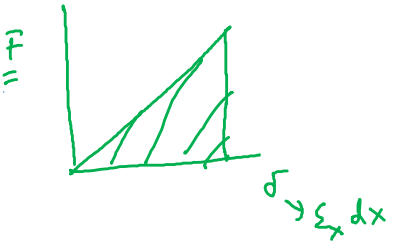
Cornell Engineering

- $\Pi = W_{int} - W_{ext}$
- $W_{ext} = P\, u_y\big]_{x=l}$
- $W_{int} = \frac{1}{2}\int_V \sigma_x \epsilon_x \, dx \, dy \, dz$
- $W_{int} = \frac{1}{2}\int_0^l EI \left(\frac{d^2 u_y}{dx^2}\right)^2 dx$



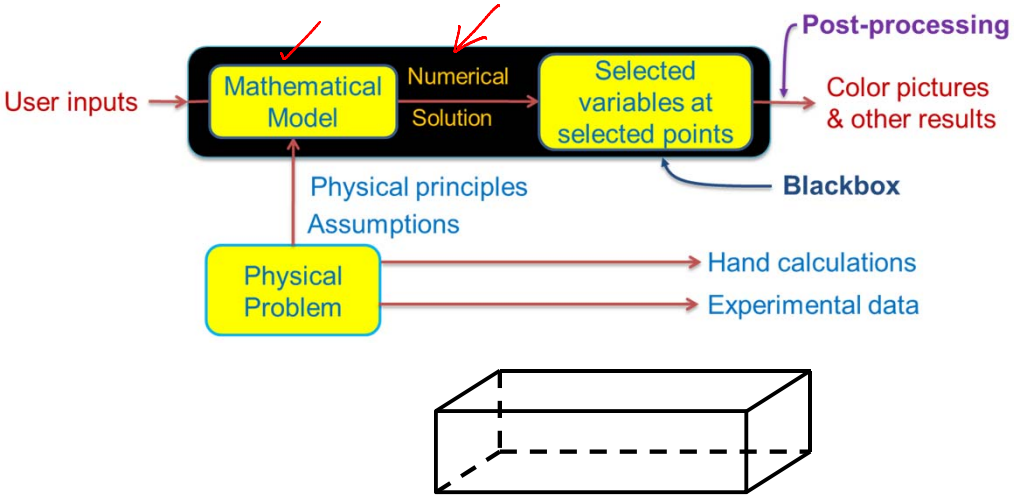






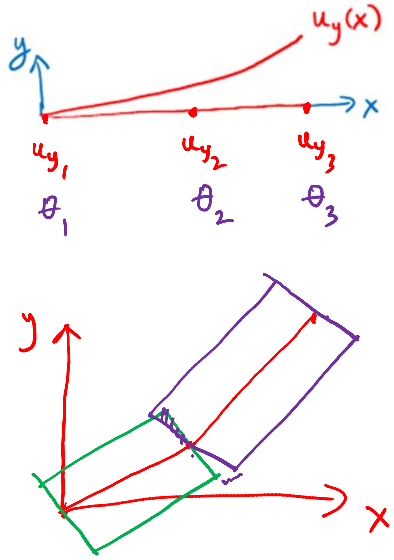
What's Under the Blackbox?

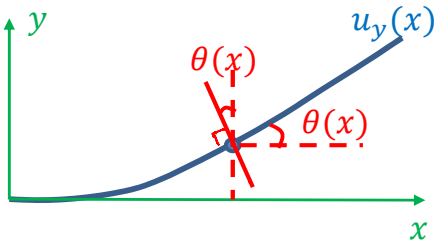
Cornell Engineering



Numerical Solution Strategy

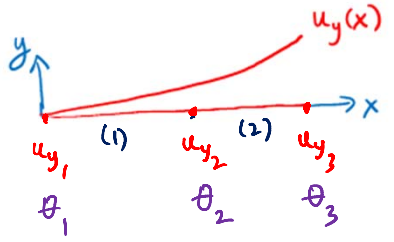
Cornell Engineering

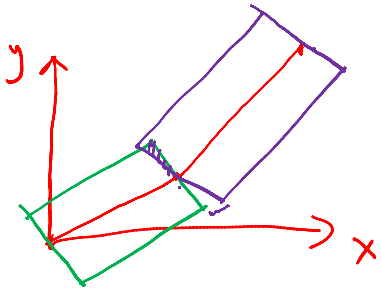




Interpolation

Cornell Engineering





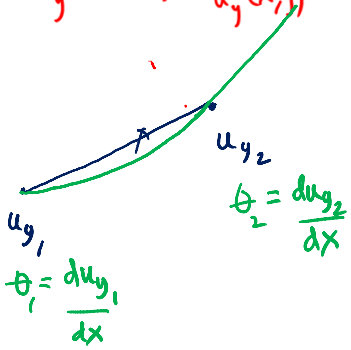
u_{y_1}
θ_1
u_{y_2}
θ_2
u_{y_3}
θ_3

Interpolation

$\rightarrow u_y(x) \rightarrow u_x(x,y)$ $u_y(x,y) \rightarrow \varepsilon_x$ $\delta_x \rightarrow \Pi$

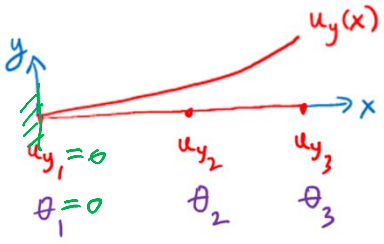
Minimize

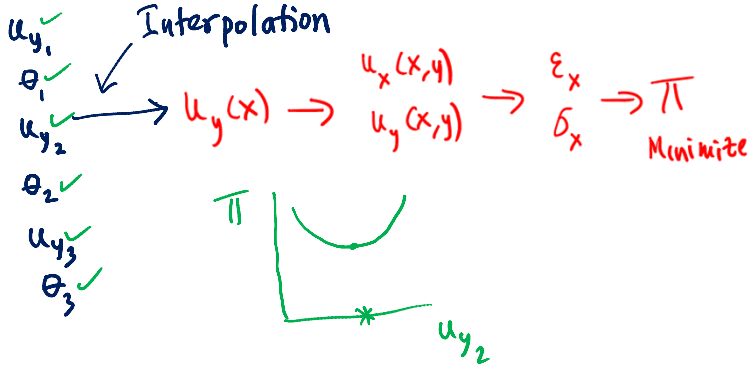
(1) $u_y(x) = A + Bx + Cx^2 + Dx^3$



Algebraic Equations Derivation

Cornell Engineering





$\frac{\partial \Pi}{\partial u_{y_2}} = 0 \Rightarrow$ Algebraic eq.

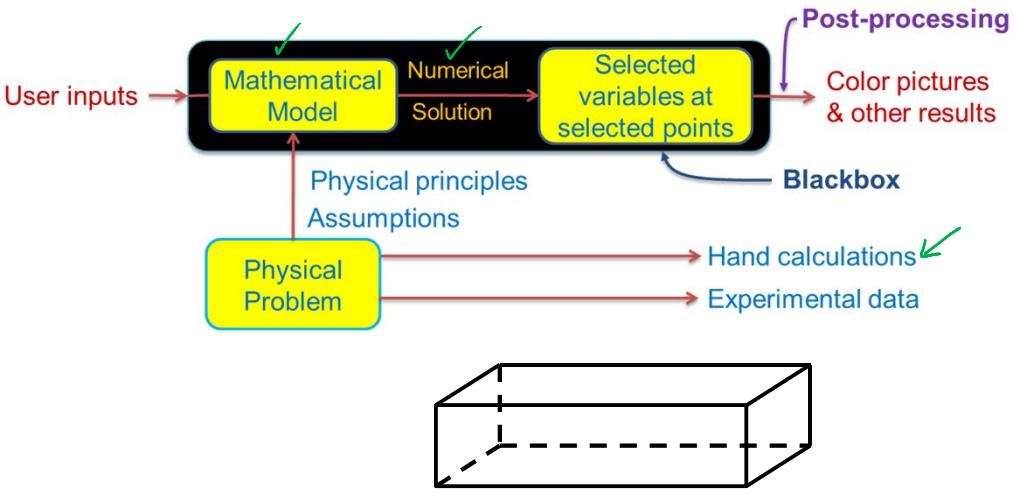
$\frac{\partial \Pi}{\partial \theta_2} = 0 \quad \frac{\partial \Pi}{\partial u_{y_3}} = 0 \quad \frac{\partial \Pi}{\partial \theta_3} = 0$

4 algebraic eqs.

4 unknowns

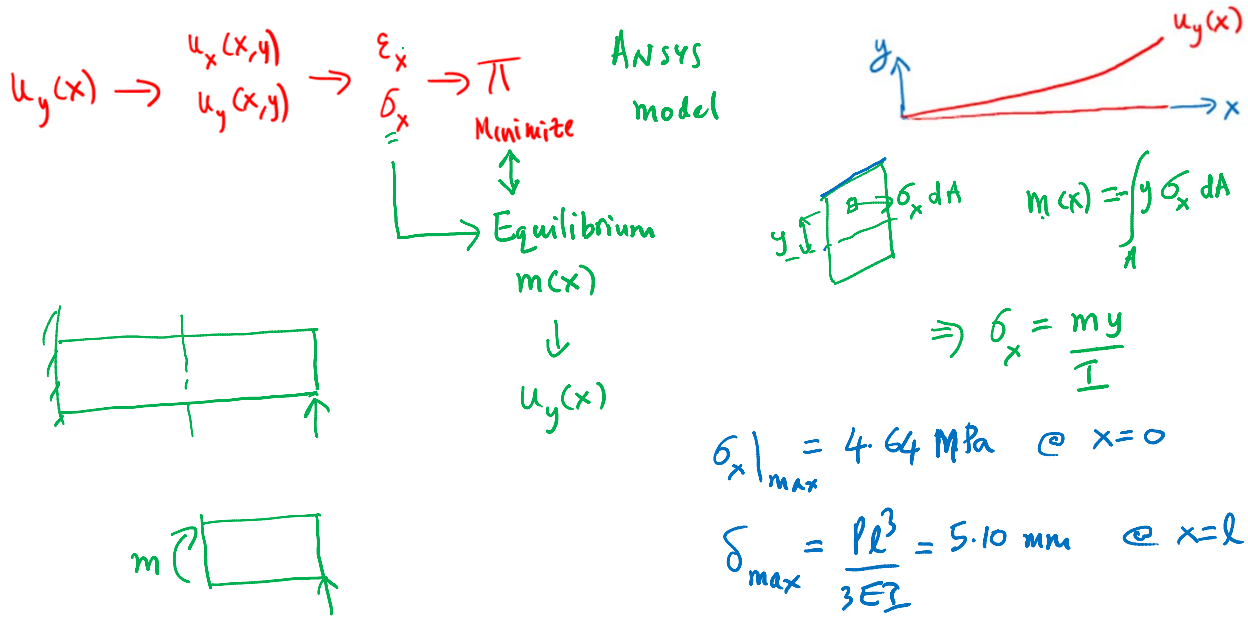
What's Under the Blackbox?

Cornell Engineering



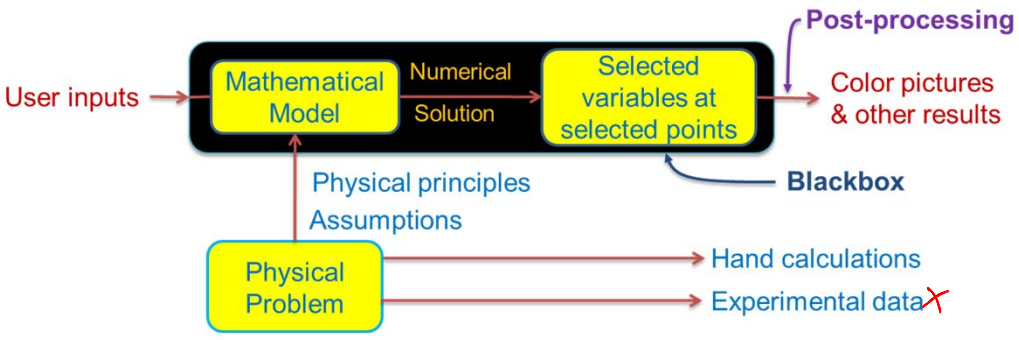
Hand Calculations

Cornell Engineering



Verification & Validation Section

Cornell Engineering



- Verification: Did I solve the model right? ✓
- Validation: Did I solve the right model? ✗

Cantilever Beam: Verification Steps

Cornell Engineering

1. Sanity checks
2. Does ANSYS solution honor the boundary conditions?
   - Zero displacement and rotation at x=0
3. Does ANSYS solution satisfy equilibrium?
   - Force and moment balance
4. How well does ANSYS solution match hand calculations?
5. Is the ANSYS solution reasonably independent of the mesh?