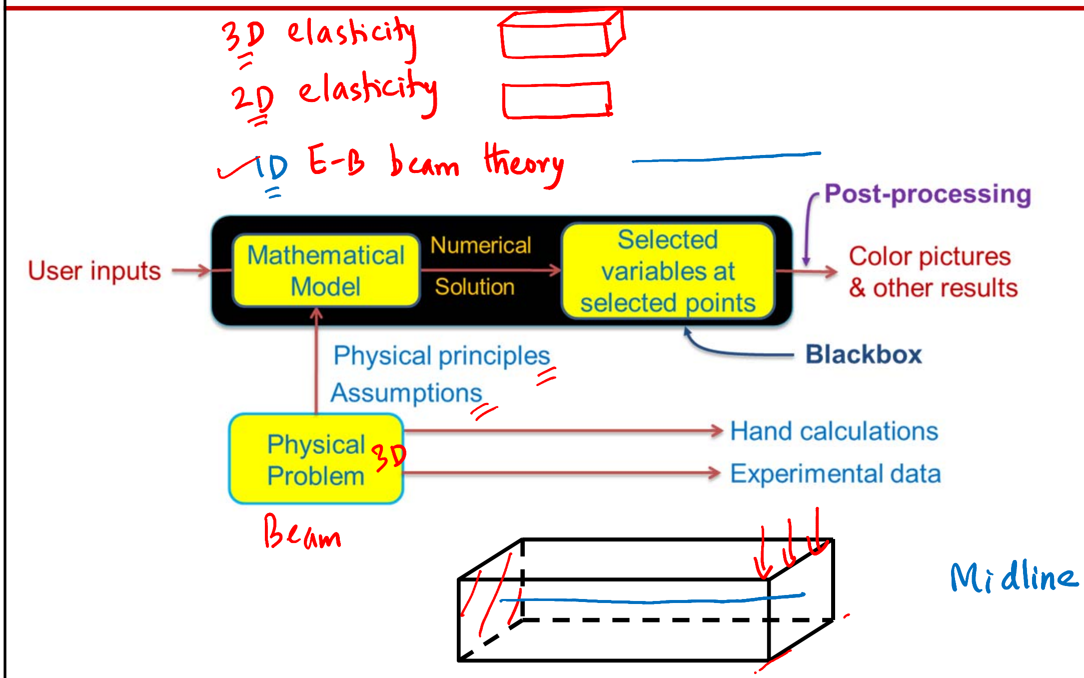
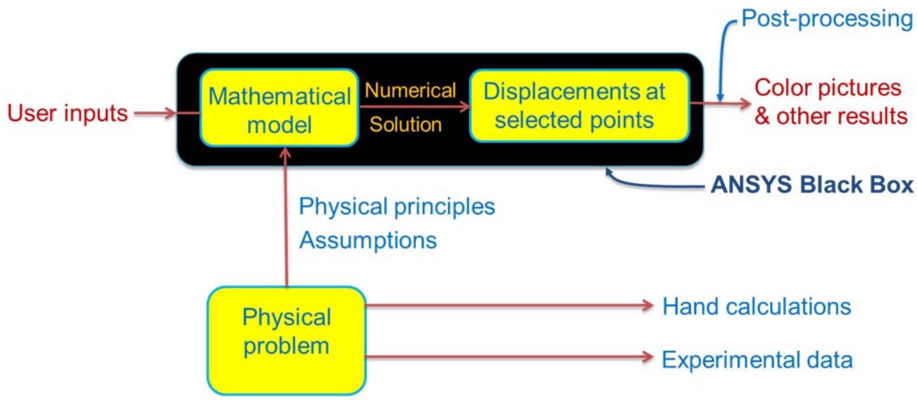
What's Under the Blackbox?



Pre-Analysis

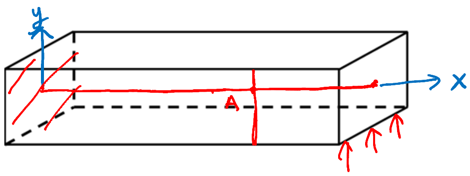


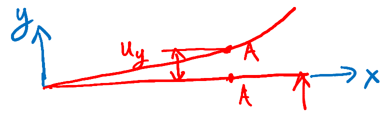
1. Mathematical model: Euler-Bernoulli beam theory
2. Numerical solution strategy: Line or beam elements
3. Hand-calculations: Max. bending stress & displacement

Euler-Bernoulli Beam Theory

Timoshenko beam theory

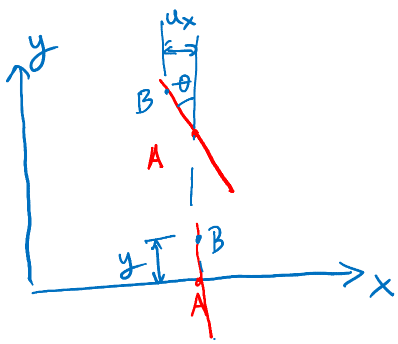
Cornell Engineering





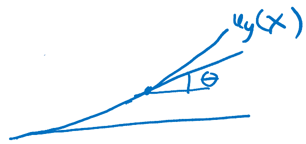
Assumptions

- No axial load
- Plane sections remain plane
- Small θ



$$u_x = -y\sin\theta \simeq -y\theta = -y\frac{du_y}{dx}$$

$$u_y \simeq u_y(x)$$

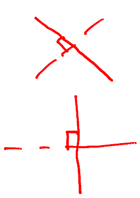


Strains and Stresses

Cornell Engineering

$$u_x \simeq -y\frac{du_y}{dx}$$

$$u_y \simeq u_y(x)$$



$$\varepsilon_x = \frac{\partial u_x}{\partial x} = -y\frac{d^2u_y}{dx^2}$$

$$\varepsilon_y = \frac{\partial u_y}{\partial y} = 0$$

$$\gamma_{xy} = 0$$

$$\sigma_x = E\varepsilon_x$$

$$\sigma_y = 0$$

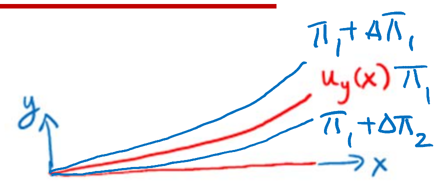
$$\tau_{xy} = 0$$

Potential Energy, Π

Cornell Engineering

$u_y(x) \rightarrow \begin{matrix} u_x(x,y) \\ u_y(x,y) \end{matrix} \rightarrow \begin{matrix} \varepsilon_x \\ \sigma_x \end{matrix} \rightarrow \Pi$

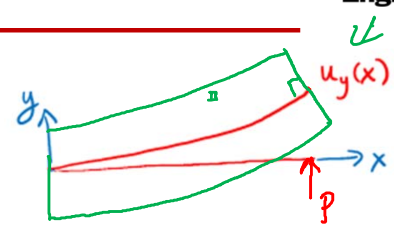
Minimize

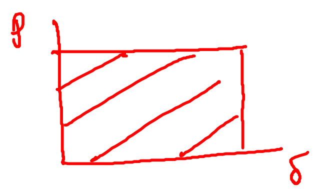


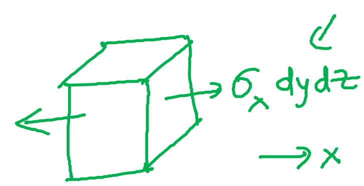
Potential Energy Minimization

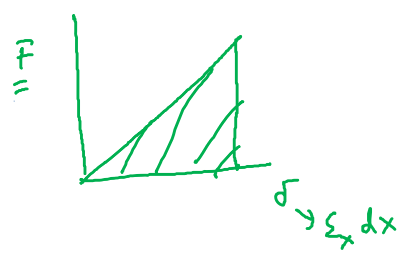
Cornell Engineering

- $\Pi = W_{int} - W_{ext}$
- $W_{ext} = P\, u_y\big]_{x=l}$
- $W_{int} = \frac{1}{2}\int_V \sigma_x \epsilon_x \, dx\, dy\, dz$
- $W_{int} = \frac{1}{2}\int_0^l EI \left(\frac{d^2 u_y}{dx^2}\right)^2 dx$









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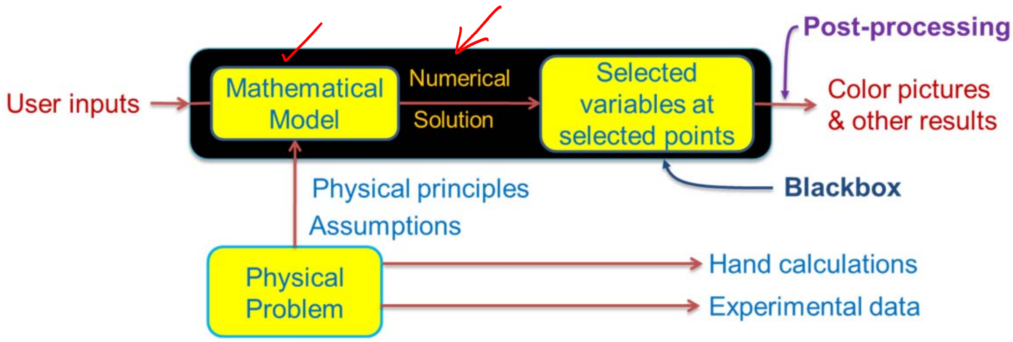
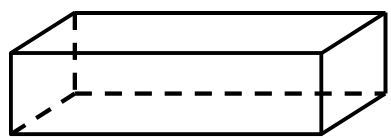
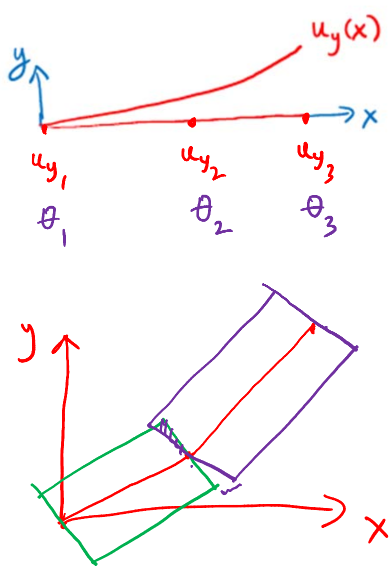
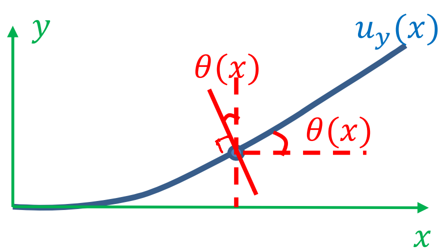


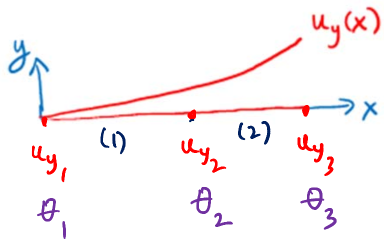


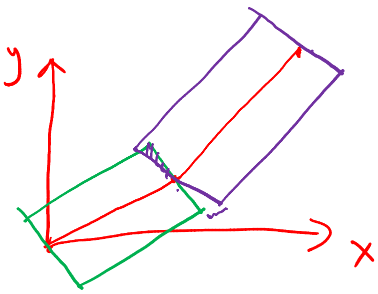
Numerical Solution Strategy





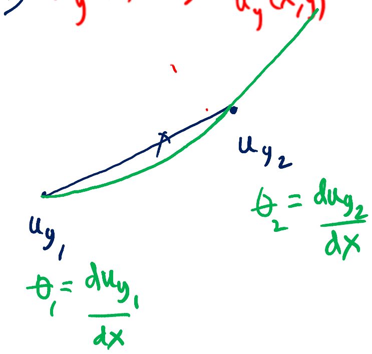
Interpolation





u_{y_1}, θ_1, u_{y_2}, θ_2, u_{y_3}, θ_3 → Interpolation → $u_y(x)$ → $u_x(x,y)$, $u_y(x,y)$ → ε_x, σ_x → Π

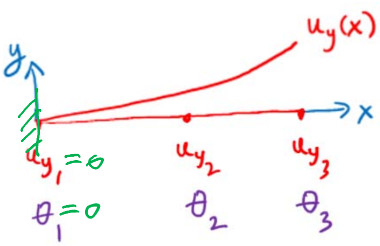
Minimize



(1) $u_y(x) = A + Bx + Cx^2 + Dx^3$

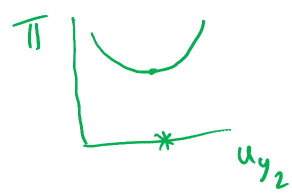
Algebraic Equations Derivation

Cornell Engineering



u_{y_1} ✓, θ_1 ✓, u_{y_2} ✓, θ_2 ✓, u_{y_3} ✓, θ_3 ✓ → Interpolation → $u_y(x)$ → $u_x(x,y)$, $u_y(x,y)$ → ε_x, σ_x → Π

Minimize



$\frac{\partial \Pi}{\partial u_{y_2}} = 0 \Rightarrow$ Algebraic eq.

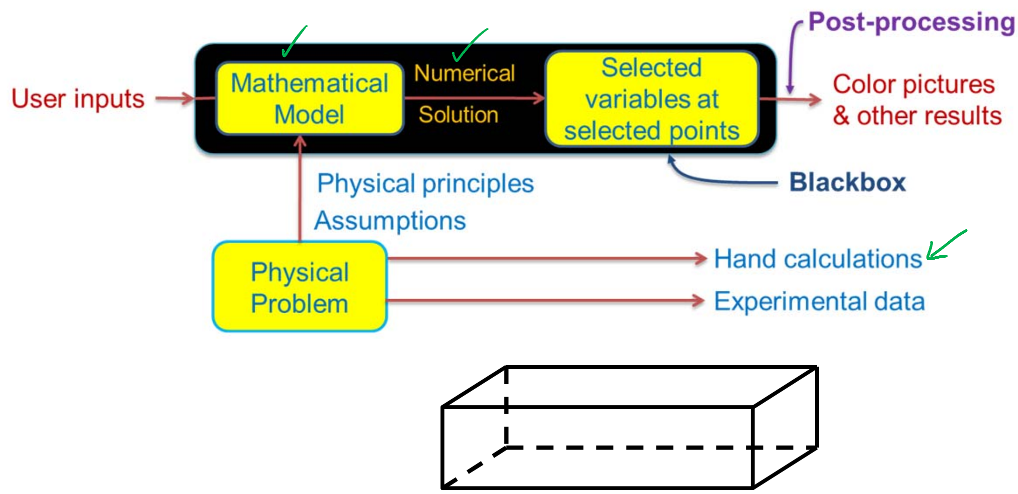
$\frac{\partial \Pi}{\partial \theta_2} = 0$ $\quad \frac{\partial \Pi}{\partial u_{y_3}} = 0$ $\quad \frac{\partial \Pi}{\partial \theta_3} = 0$

4 algebraic eqs.

4 unknowns

What's Under the Blackbox?

Cornell Engineering



Hand Calculations

Cornell Engineering

$u_y(x) \Rightarrow u_x(x,y),\ u_y(x,y) \Rightarrow \varepsilon_x,\ \sigma_x \Rightarrow \Pi$

Minimize

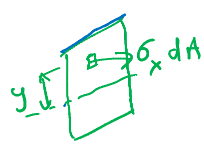
$\Updownarrow$

Equilibrium $m(x)$

$\Downarrow$

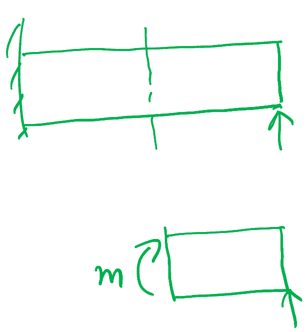
$u_y(x)$

ANSYS model



$m(x) = -\int_A y\,\sigma_x\,dA$

$\Rightarrow \sigma_x = \frac{my}{I}$



$\sigma_x\big|_{max} = 4.64$ MPa @ $x=0$

$\delta_{max} = \frac{Pl^3}{3EI} = 5.10$ mm @ $x=l$