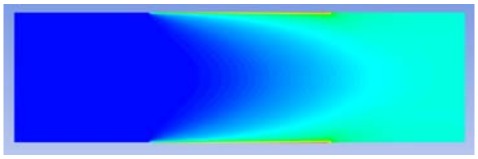

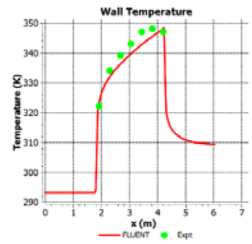

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Computational Fluid Dynamics (CFD) Simulation for HT2 Experiment

MAE 4272

Mechanical & Aerospace Engineering

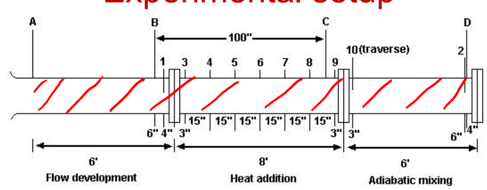
Cornell University

Cornell Engineering

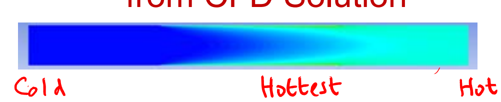
Computational Fluid Dynamics (CFD)

- CFD software can simulate flow behavior by solving the governing equations of fluid flow numerically
  - CFD solution is *approximate*
  - CFD software we'll use: ANSYS Fluent™
- Benefits
  - Can visualize the flow and do what-if studies
- Challenges
  - Garbage in, garbage out
  - Need to determine carefully how good the results are

Experimental setup



Temperature distribution from CFD Solution



Verification and Validation (V&V)

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- Systematic process for checking results
- Each of these terms has a specific meaning
  - More on that soon
- To understand how to verify and validate results, need to know what's inside the CFD blackbox

What's Inside the CFD Blackbox?

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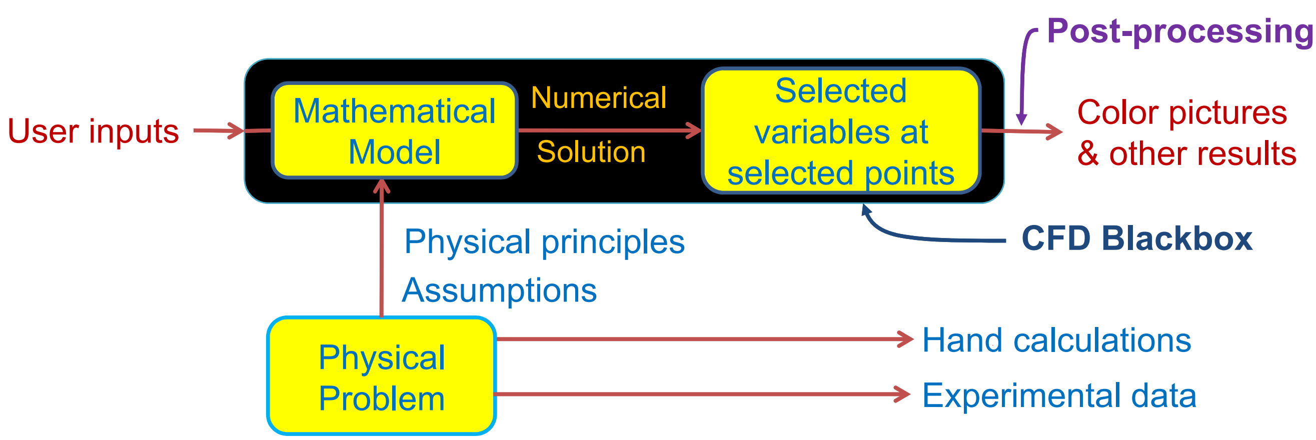

Verification & Validation: Definition

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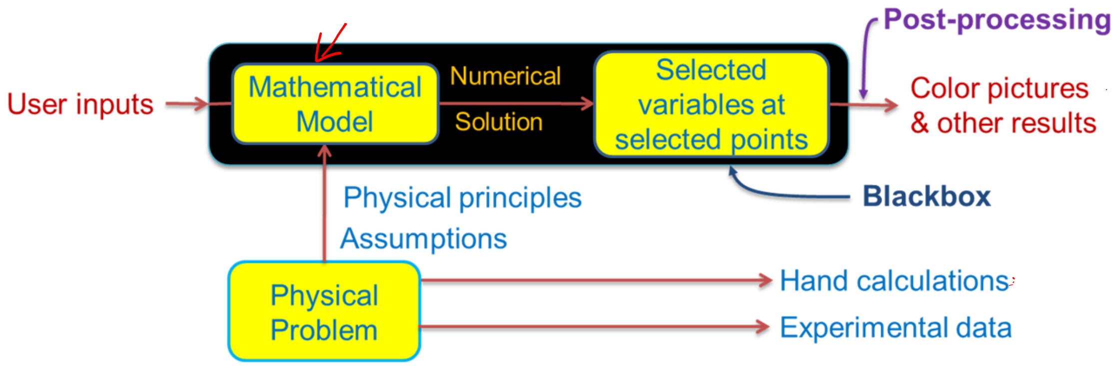

- Verification: Did I solve the model right?
  - Check consistency with mathematical model, level of numerical errors, comparison with hand calcs
- Validation: Did I solve the right model?
  - Check against experimental data

Pre-Analysis: Forms the Basis for V&V

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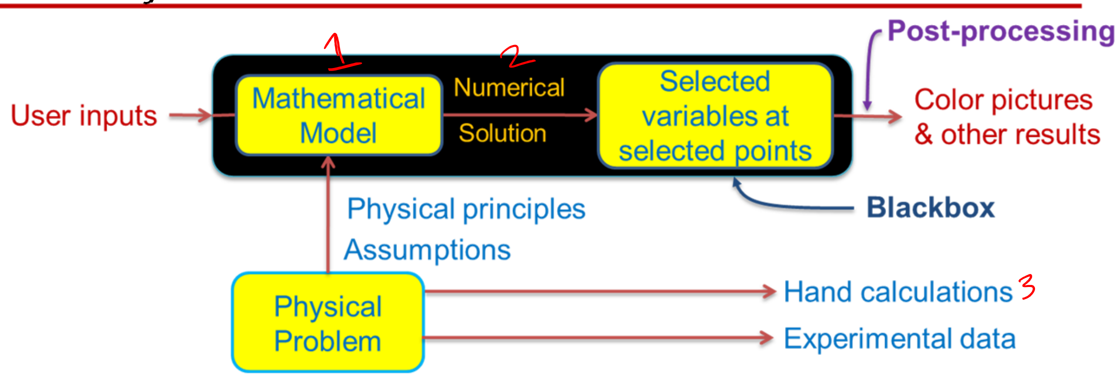

1. Mathematical model
2. Numerical solution procedure
3. Hand-calculations of expected results/trends

Mathematical Model: Boundary Value Problem

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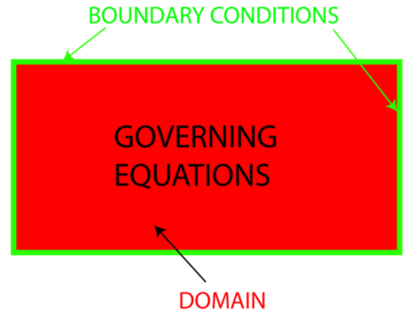
- *Governing eqs.* defined in a *domain*
- *Boundary conditions* defined at the edges of the *domain*
- Governing eqs. are based on conservation of mass, momentum and energy applied to a differential fluid blob
- Governing eqs. are very complicated non-linear differential equations
  - Variable density
  - Turbulent flow
- We'll start by looking at constant density equations and then move to variable density equations with turbulence effects



Mathematical Model: Axisymmetric Assumption

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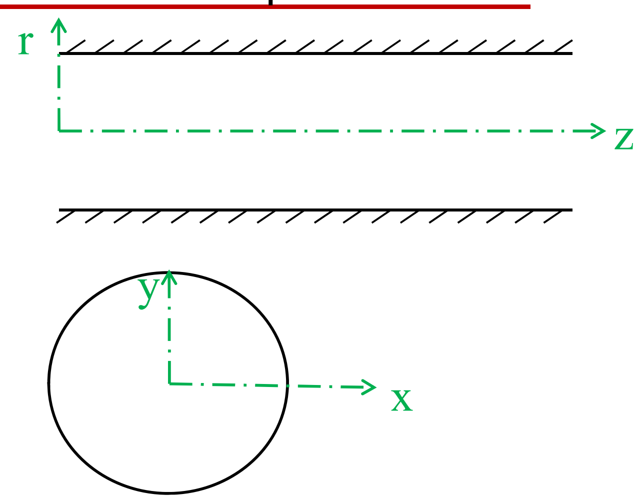
- Use cylindrical co-ordinates (r, θ, z)
- $p = p(r, z)$
- $\vec{\boldsymbol{V}} = v_r\,\hat{e}_r + v_z\,\hat{e}_z +$ ~~$v_\theta\,\hat{e}_\theta$~~
- $v_r = v_r(r, z)$
- $v_z = v_z(r, z)$



Domain

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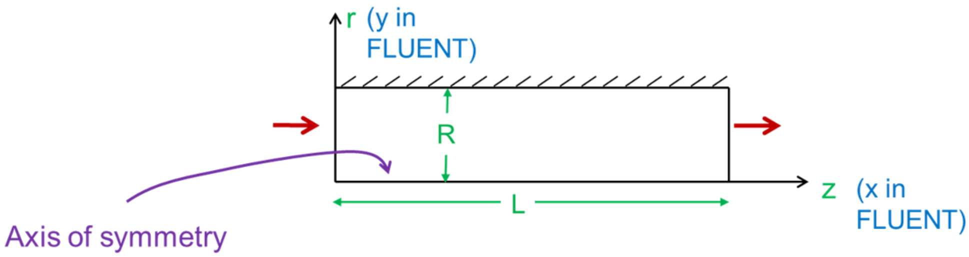

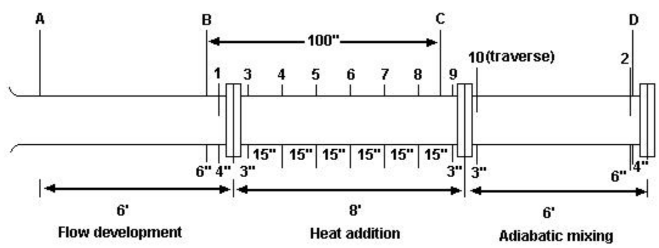

Length of pipe included in the simulation: From A to D

Governing Equations for Constant Density Flows

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1. Conservation of mass
   - $\nabla \cdot \vec{V} = 0$
2. Conservation of momentum ($\vec{F} = m\vec{a}$ in axial and radial directions)
   - $\rho(\vec{V} \cdot \nabla)\vec{V} = -\nabla p + \mu \nabla^2 \vec{V}$
3. Conservation of energy (First law of thermodynamics)
   - $\rho(\vec{V} \cdot \nabla)(C_V T) = k\nabla^2 T - p(\nabla \cdot \vec{V}) + \mu\Phi$

4 unknown functions:

$v_r(r,z), v_z(r,z), p(r,z), T(r,z)$

Energy equation is decoupled from mass and momentum eqs.

Governing Equations for *Variable* Density Flows

Cornell Engineering

1. Conservation of mass
   - $\nabla \cdot (\rho \vec{V}) = 0$
2. Conservation of momentum ($\vec{F} = m\vec{a}$ in axial and radial directions)
   - $\rho(\vec{V} \cdot \nabla)\vec{V} = -\nabla p + \mu \nabla \cdot (\nabla \vec{V} + \nabla \vec{V}^T) - \frac{2}{3}\mu \nabla(\nabla \cdot \vec{V})$
3. Conservation of energy (First law of thermodynamics)
   - $\rho(\vec{V} \cdot \nabla)(C_V T) = k\nabla^2 T - p\,(\nabla \cdot \vec{V}) + \mu\Phi$
4. Ideal gas law (user input)
   - $\rho = \frac{p}{RT} \simeq \frac{p_{average}}{RT}$

5 unknown functions:

$v_r(r,z), v_z(r,z), p(r,z), T(r,z), \rho(r,z)$

Energy equation is coupled to mass and momentum eqs.

Turbulence Overview

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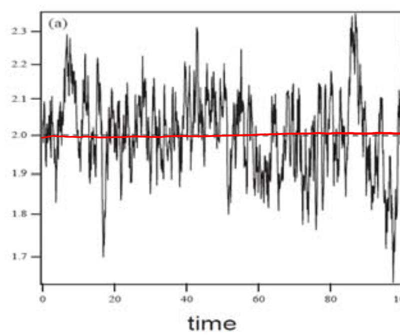
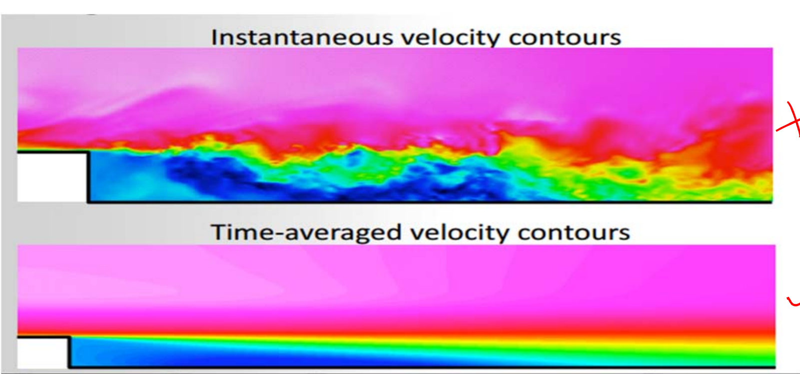
- Turbulent flow: Fluctuating but not about the laminar solution
- Reynolds decomposition:
  - $v_z = \bar{v}_z + v_z'$





From ANSYS training documentation. Used courtesy of ANSYS, Inc.

Reynolds-Averaged Governing Equations

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1. Conservation of mass
   - $\nabla \cdot (\bar{\rho}\,\bar{\vec{V}}) = 0$
2. Conservation of momentum ($\vec{F} = m\vec{a}$ in axial and radial directions)
   - $\bar{\rho}(\bar{\vec{V}} \cdot \nabla)\bar{\vec{V}} = -\nabla\bar{p} + \mu\,\nabla \cdot (\nabla\,\bar{\vec{V}} + \nabla\,\bar{\vec{V}}^T) - \frac{2}{3}\mu\,\nabla(\nabla \cdot \bar{\vec{V}})$ + Turbulent terms
3. Conservation of energy (First law of thermodynamics)
   - $\rho(\bar{\vec{V}} \cdot \nabla)(C_V\bar{T}) = k\nabla^2\bar{T} - p\,(\nabla \cdot \bar{\vec{V}}) + \mu\bar{\Phi}$ + Turbulent terms
4. Ideal gas law
   - $\bar{\rho} = \frac{\bar{p}}{R\bar{T}} \simeq \frac{p_{average}}{R\bar{T}}$

Turbulent terms depend on unknown fluctuating quantities v_z'etc.

Can calculate approximately using a turbulence model

$k - \epsilon$ Turbulence Model

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- k: Turbulent kinetic energy
  - Measure of how much energy is contained in the fluctuations
- ϵ: Turbulent dissipation
  - Measure of the rate at which turbulent kinetic energy is dissipated
- Two additional conservation equations: one each for k and ϵ
- Unknown turbulent terms are calculated from k and ϵ

$\bar{\rho}(\bar{\vec{V}} \cdot \nabla)\bar{\vec{V}} = -\nabla p + \mu\,\nabla \cdot (\nabla\,\vec{V} + \nabla\,\vec{V}^T) - \frac{2}{3}\mu\,\nabla(\nabla \cdot \vec{V})$
+ Turbulent terms k, ϵ

Reynolds-Averaged Governing Eqs. with $k - \epsilon$ Model

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1. Conservation of mass
   - $\nabla \cdot (\bar{\rho}\, \vec{\bar{V}}) = 0$
2. Conservation of momentum ($\vec{F} = m\vec{a}$ in axial and radial directions)
   - $\bar{\rho}(\vec{\bar{V}} \cdot \nabla)\vec{\bar{V}} = -\nabla \bar{p} + \mu\, \nabla \cdot (\nabla \vec{\bar{V}} + \nabla \vec{\bar{V}}^T) - \frac{2}{3}\mu\, \nabla(\nabla \cdot \vec{\bar{V}})$ + Turbulent terms
3. Conservation of energy (First law of thermodynamics)
   - $\rho(\vec{\bar{V}} \cdot \nabla)(C_V \bar{T}) = k\nabla^2 \bar{T} - p\,(\nabla \cdot \vec{\bar{V}}) + \mu\bar{\Phi}$ + Turbulent terms
4. Ideal gas law
   - $\bar{\rho} = \frac{\bar{p}}{RT} \simeq \frac{p_{average}}{RT}$
5. k conservation eq.
6. ϵ conservation eq.

7 unknown functions: $\bar{v}_r, \bar{v}_z, \bar{p}, \bar{T}, \bar{\rho}, \mathrm{k}, \epsilon$

6 differential eqs.
+ 1 algebraic eq.

Mathematical Model: Boundary Conditions

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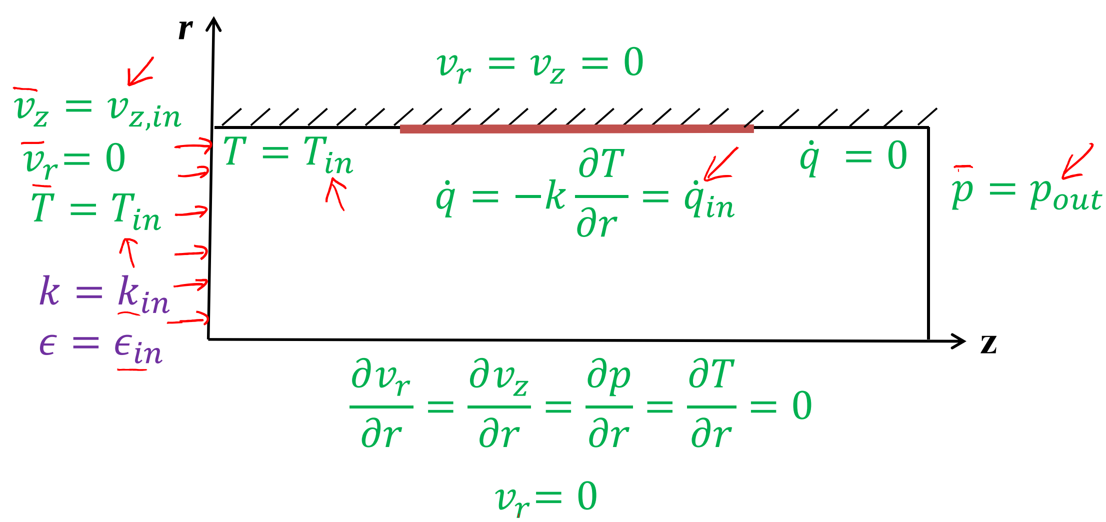

Pre-Analysis: Forms the Basis for V&V

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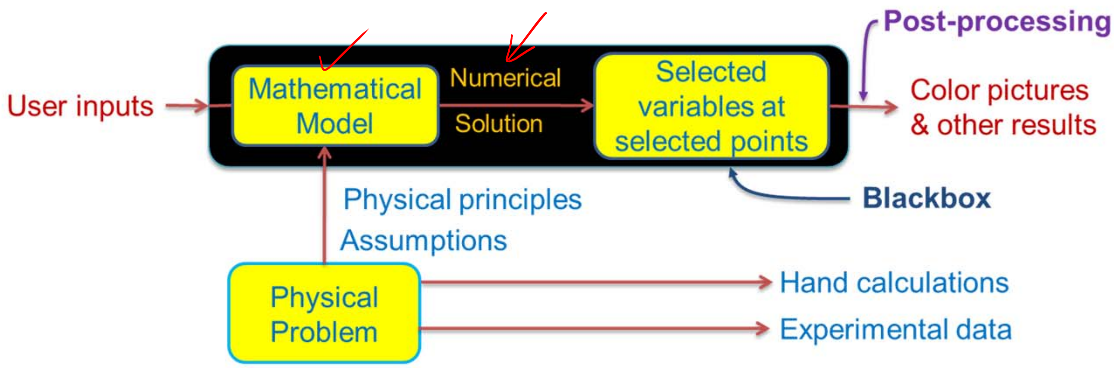

1. Mathematical model
2. Numerical solution procedure
3. Hand-calculations of expected results/trends

Numerical Solution Procedure: Finite Volume Method

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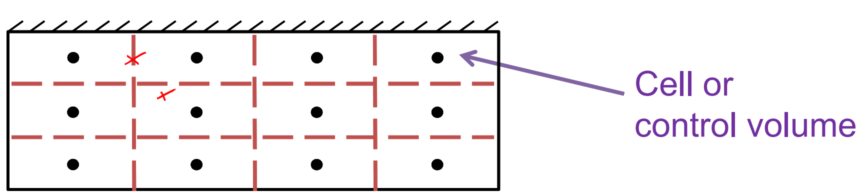
- Divide the domain into multiple control volumes or "cells"
- Reduce the problem to determining $v_r, v_z, p, T, \rho, \mathrm{k}, \epsilon$ at selected points (cell centers)
  - "Discretization"
- Use interpolation to determine variables away from cell centers



How to Find Velocity, Pressure etc. at Cell Centers?

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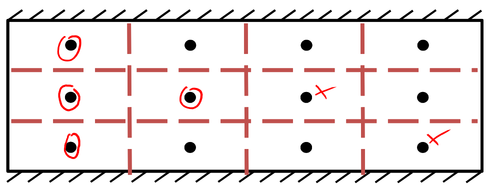
Mathematical Model (Boundary Value Problem) → System of algebraic equations in cell-center values → Invert → Cell-center values of $v_r, v_z, p, T, \rho, \mathrm{k}, \epsilon$ → Post processing → $p(r,z), v_z(r,z)$, Wall shear etc.

Control volume balance for each cell

Each algebraic equation will relate a cell-center value to its neighbors



Discretization and Linearization: Overview

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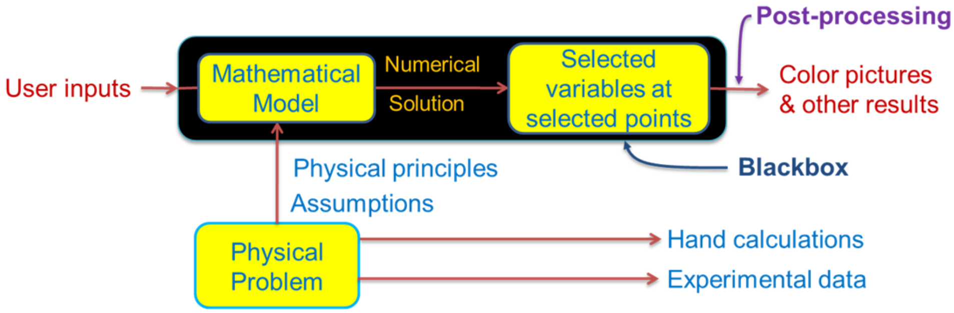
BVP

Differential form of governing equations + BCs → Integral form of governing equations + BCs → Discretization error → Algebraic equations relating cell-center values → Linearization error → Linearized algebraic equations relating cell-center values → Solve iteratively updating guess after each iteration → Stop iterations when imbalances are below tolerance

Verification Steps



Pre-Analysis Steps

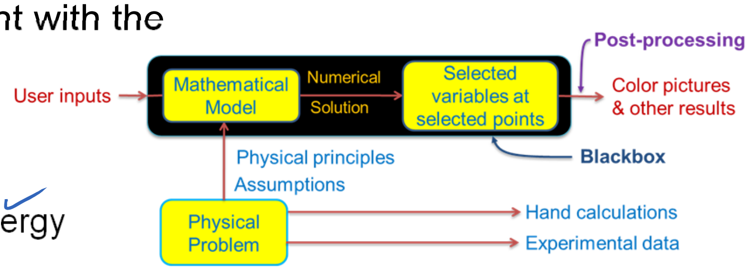
1. Mathematical model
2. Numerical solution procedure
3. Hand-calculations of expected results/trends

Verification Steps

1. Results consistent with mathematical model?
2. Numerical errors acceptable?
3. Results compare well with hand calcs?

Verification Checklist

1. Are the CFD results consistent with the math model?
   - Check BCs
   - Check material properties
   - Check coordinate system
   - Check mass, momentum & energy conservation
   - Check density-temperature coupling
   - Check sensitivity to turbulence model
2. Are numerical errors acceptable?
   - Check linearization error by monitoring imbalances and drag coefficient
   - Check discretization error by refining mesh
3. Do the CFD results compare reasonably well with hand calculations?



Reynolds-Averaged Governing Eqs. with $k - \epsilon$ Model

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$\bar{v}_r, \bar{v}_z, \bar{p}, \bar{T}, \bar{\rho}, \mathrm{k}, \epsilon$

6 differential eqs.
+ 1 algebraic eq.

Validation Steps

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1. Comparison with measurements from current experiment
2. Comparison with correlations from classical experiments